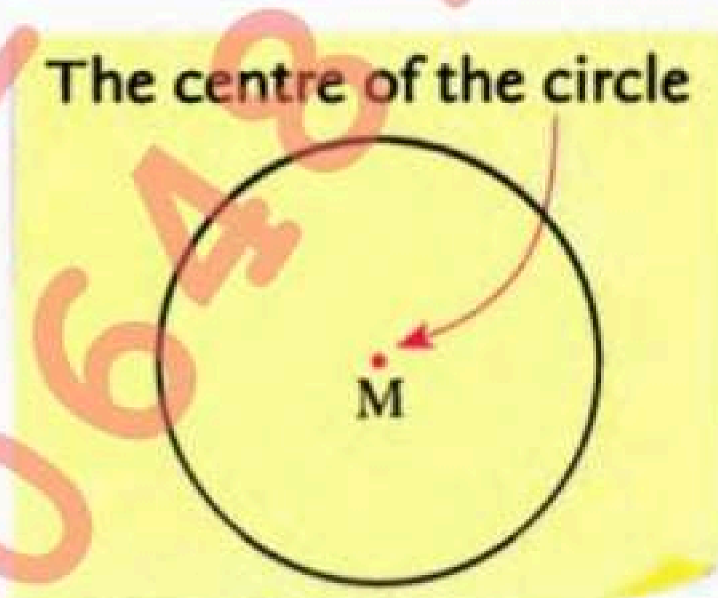
Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [1] : Basic Definitions And Concepts

The circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

- The fixed point is called "**the centre of the circle**".
- The constant distance is called "the radius length of the circle".
- The circle is usually denoted by its centre , so we say the circle M to mean the circle whose centre is the point M



Partition of the plane by the circle

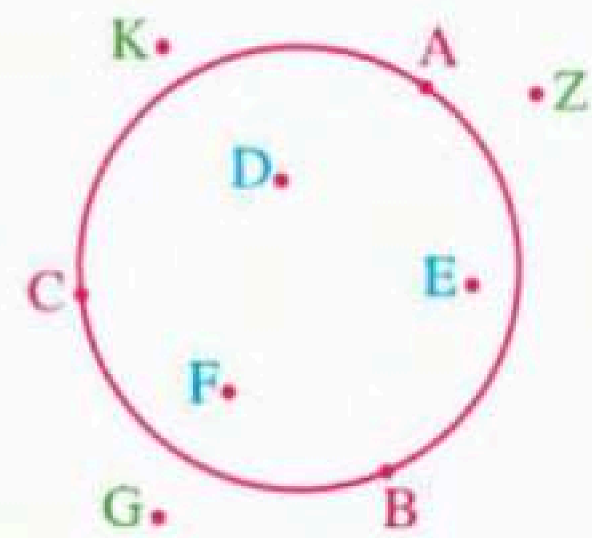
Any circle divides the plane into three sets of points which are :

1. The set of points of the circle.
2. The set of points inside the circle.
3. The set of points outside the circle.

For example :

The drawn circle in the opposite figure divides the plane into :

1. The set of points of the circle «on the circle» as : A , B , C , ...
2. The set of points inside the circle as : D , E , F , ...
3. The set of points outside the circle as : Z , K , G , ...

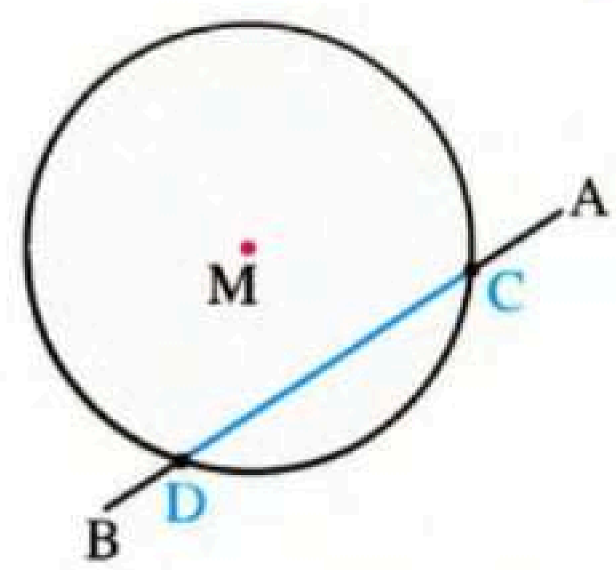


The surface of the circle is : the set of points of the circle $\cup$ the set of points inside it.

So , the surface of the circle differs from the circle.

For example:

In the opposite figure :

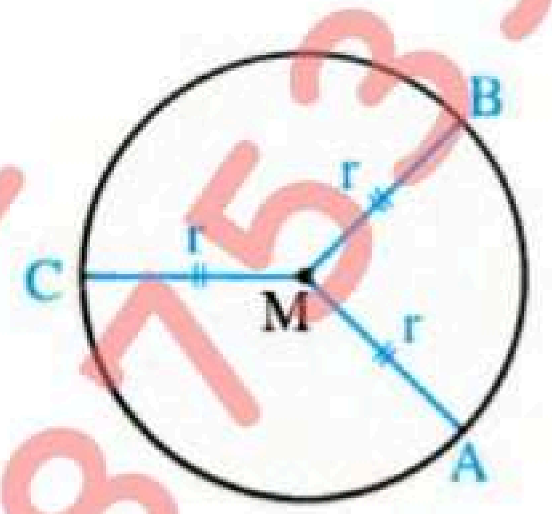


- $\overline{AB} \cap$ the circle = $\{C , D\}$ but $\overline{AB} \cap$ the surface of the circle = $\overline{CD}$
- $M \notin$ the circle but $M \in$ the surface of the circle.

The radius of the circle

It is a line segment whose endpoints are the centre of the circle and any point on the circle.

In the opposite figure :



If the points A , B and C belong to the circle M ,

then $\overline{MA}$, $\overline{MB}$ and $\overline{MC}$ are called radii of the circle M

and MA = MB = MC = r (where r is the radius length of the circle).

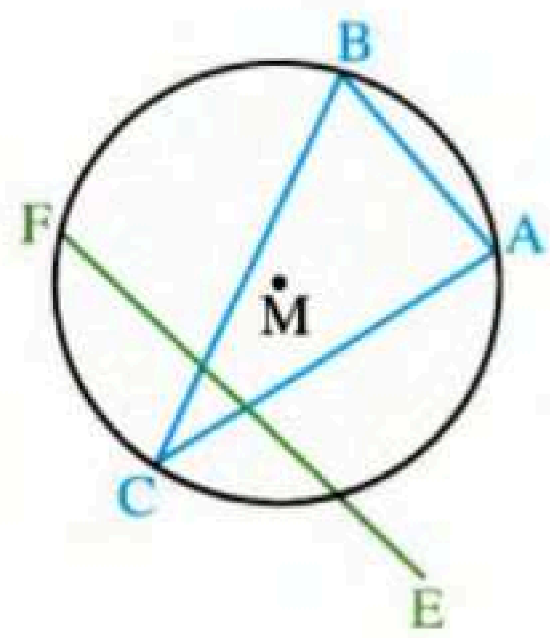
Notice that :

1. Any circle has an infinite number of radii and all of them are equal in length.
2. If two radii of two circles are equal in length , then the two circles are congruent and vice versa.

The chord of the circle

It is a line segment whose endpoints are any two points on the circle.

In the opposite figure :



If A , B and C belong to the circle M ,

then each of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$

is a chord of the circle M

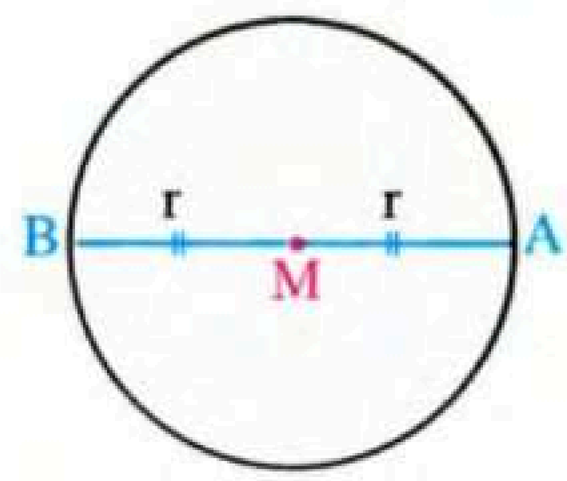
Notice that :

$\overline{EF}$ is not a chord of the circle M because E $\notin$ the circle M

The diameter of the circle

It is a chord passing through the centre of the circle.

In the opposite figure :



If M is a circle , $\overline{AB}$ is a chord of it

, M $\in \overline{AB}$, then $\overline{AB}$ is a diameter of the circle M

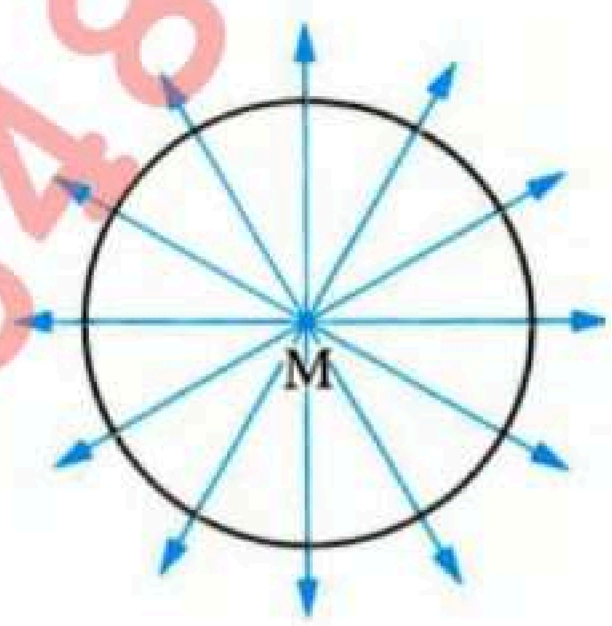
Notice that :

1. Any circle has an infinite number of diameters and all of them are equal in length.
2. The diameter of the circle is the longest chord of the circle , and its length = 2 r

The circumference of the circle and its area

- The circumference of the circle = $2 \pi r$
- The area of the circle = πr^2

Symmetry in the circle



- Any straight line passing through the centre of the circle is an axis of symmetry of it.
- Since the number of these straight lines are infinite , then the circle has an infinite number of axes of symmetry.

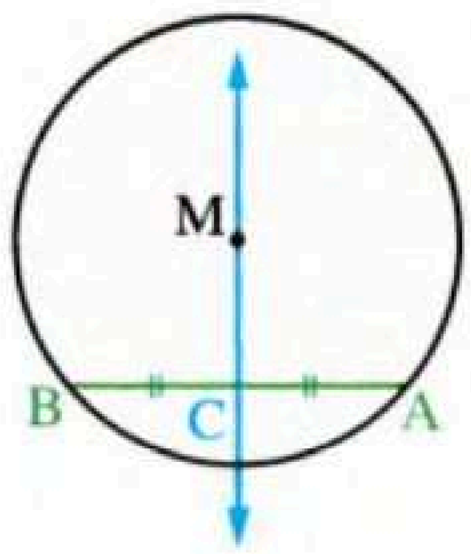
Important corollaries

Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it is perpendicular to this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and C is the midpoint of $\overline{AB}$, then $\overleftrightarrow{MC} \perp \overline{AB}$

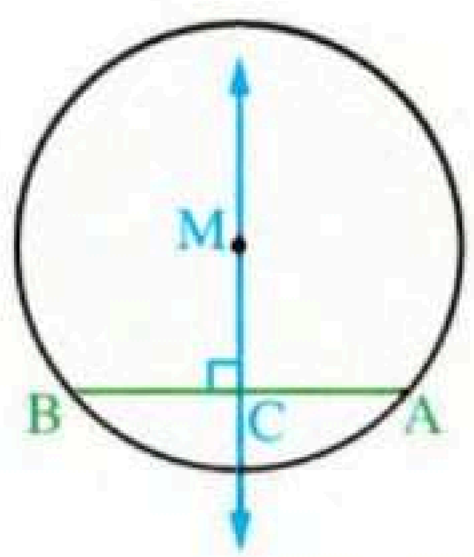


Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then C is the midpoint of $\overline{AB}$



Corollary 3

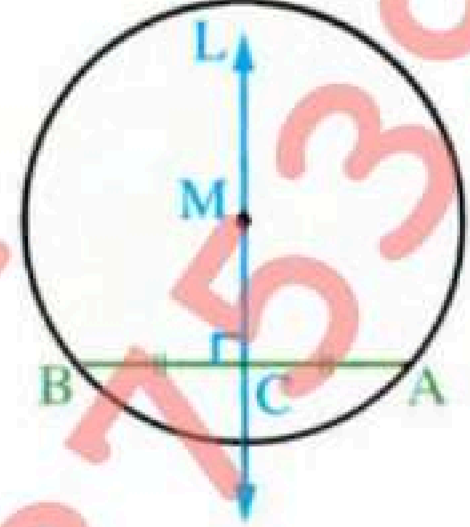
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

and the straight line L $\perp$ $\overline{AB}$ from the point C ,

then M $\in$ the straight line L



From the previous, we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

Exercises

[A] Essay problems : -

1 **In the opposite figure :**

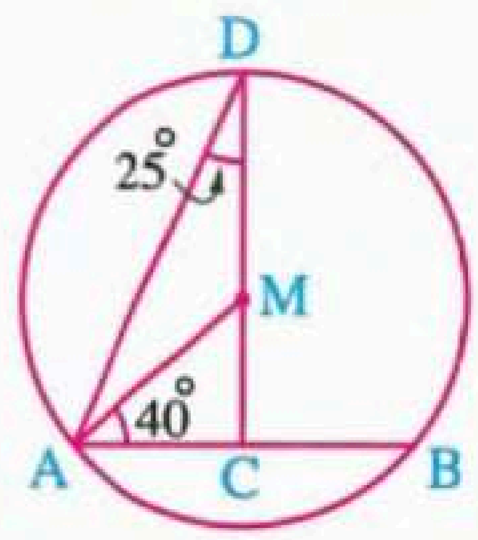
$\overline{AB}$ is a chord of the circle M ,

m ($\angle$ D) = 25°

and m ($\angle$ MAC) = 40°

Prove that :

C is the midpoint of $\overline{AB}$



(Kafr El-Sheikh 09)

2 **In the opposite figure :**

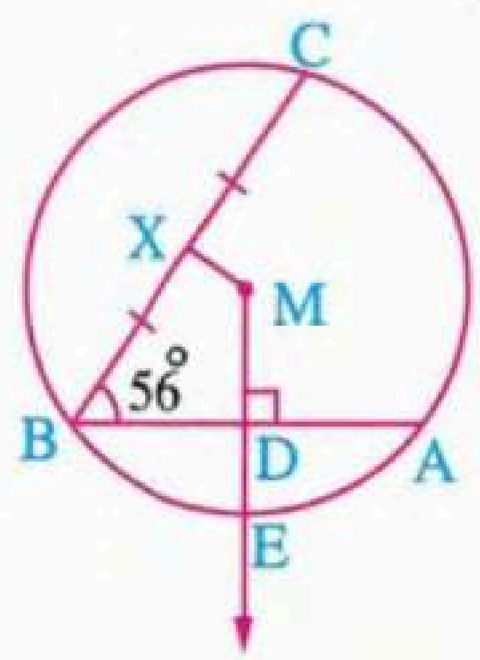
$\overline{AB}$ and $\overline{BC}$ are two chords in circle M ,

which has radius length of 5 cm.,

$\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and inersects the circle M at E ,

X is the midpoint of $\overline{BC}$, AB = 8 cm., m ($\angle$ ABC) = 56°

Find : 1 m ($\angle$ DMX) 2 The length of $\overline{DE}$



(El-Menia 19 , El-Gharbia 17 , Souhag 15) « 124° , 2 cm. »

3

In the opposite figure :

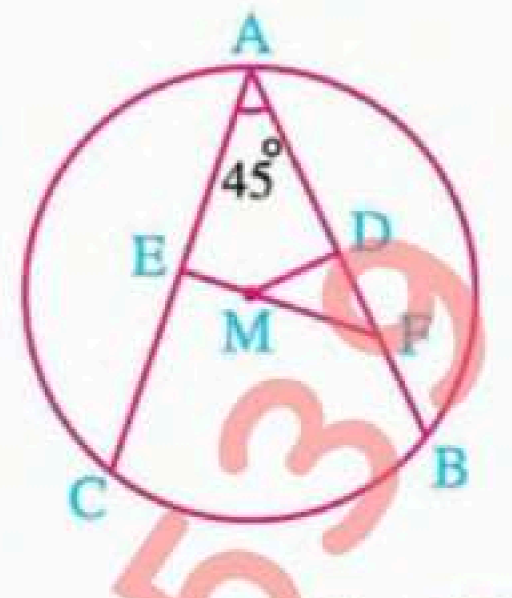
$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M ,

m (∠ BAC) = 45° ,

D and E are the midpoints

of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : Δ DFM is an isosceles triangle.



(New Valley 05)

4

In the opposite figure :

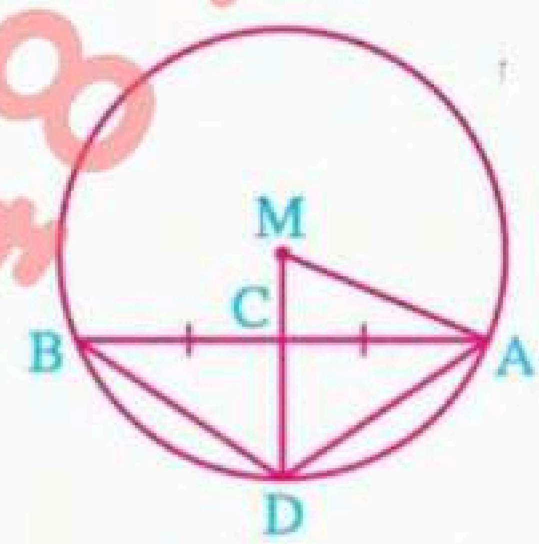
M is a circle of radius length 13 cm. ,

$\overline{AB}$ is a chord of length 24 cm. ,

C is the midpoint of $\overline{AB}$

and $\overrightarrow{MC}$ ∩ circle M = {D}

Find : The area of the triangle ADB



(El-Dakahlia 13) « 96 cm^2. »

5

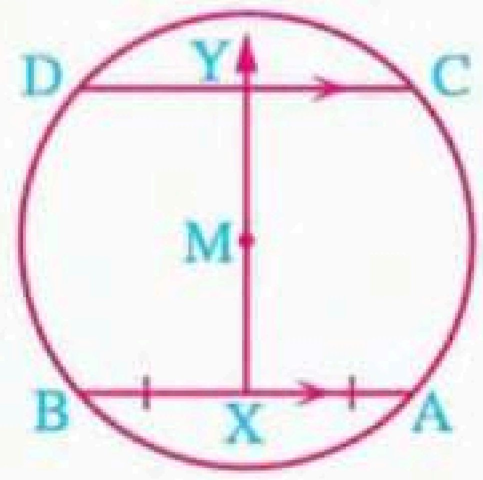
In the opposite figure :

M is a circle , $\overline{AB}$ // $\overline{CD}$,

X is the midpoint of $\overline{AB}$

and $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



(El-Menia 18 , Assiut 18 , Aswan 15 , Alexandria 13)

6

In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in circle M

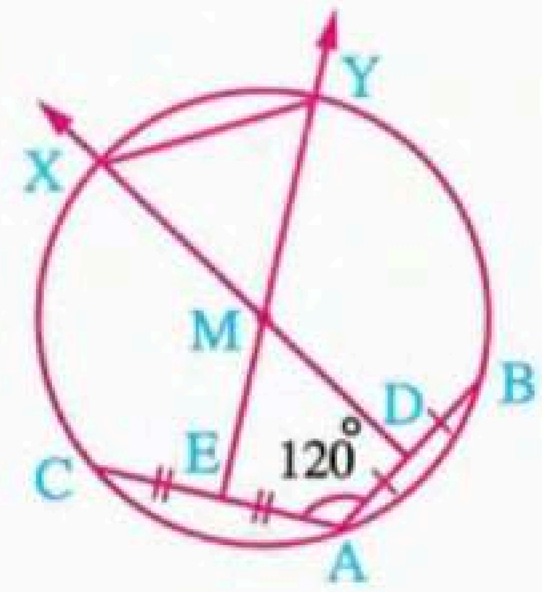
that includes an angle of measure 120° ,

D and E are the two midpoints of $\overline{AB}$ and $\overline{AC}$

respectively , $\overrightarrow{DM}$ and $\overrightarrow{EM}$ are drawn to intersect

the circle at X and Y respectively.

Prove that : The triangle XYM is an equilateral triangle.



(Aswan 16 , Beni Suef 15)

7

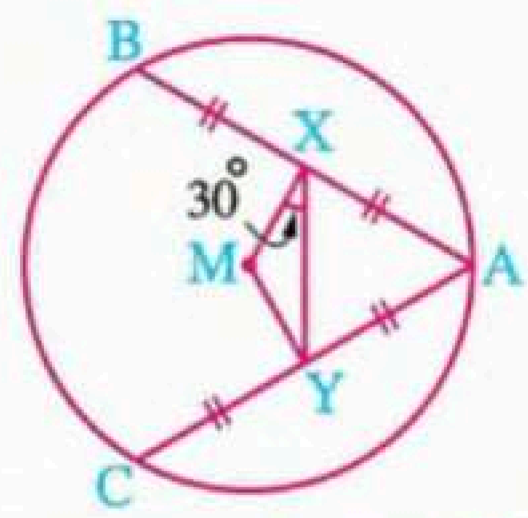
In the opposite figure :

AC = AB , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$,

m (∠ MXY) = 30°

Prove that : The triangle AXY is equilateral.



(Assiut 14)

8

In the opposite figure :

Two concentric circles with centre M ,

$\overline{AB}$ is a chord of the greater circle

and intersects the smaller circle at C , D

and $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

(El-Gharbia 18 , Qena 18 , Qena 17 , Red Sea 12)

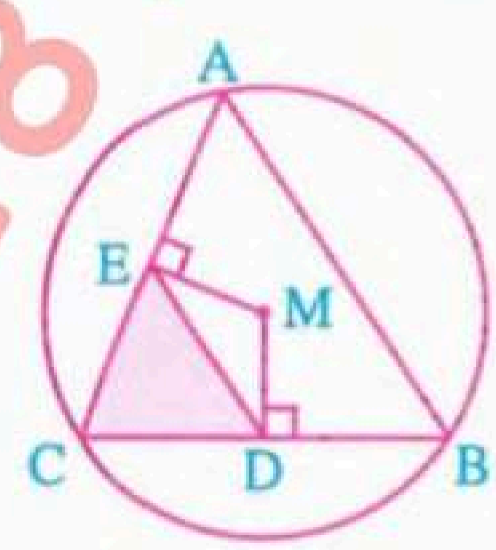
9

In the opposite figure :

ABC is a triangle drawn inside a circle with centre M (inscribed triangle) , $\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$

Prove that :

1. $\overline{ED} \mathbin{//} \overline{AB}$
2. The perimeter of Δ CDE = $\frac{1}{2}$ the perimeter of Δ ABC



(Kafr El-Sheikh 16 , El-Beheira 13)

10

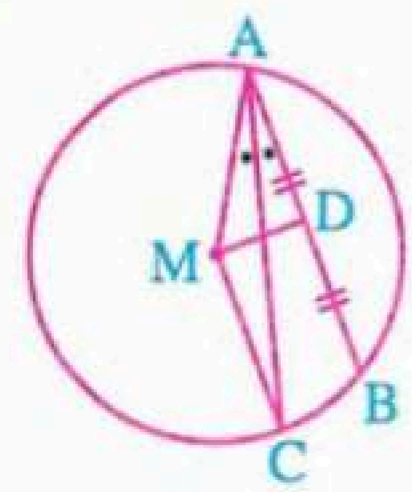
In the opposite figure :

$\overline{AB}$ is a chord of circle M ,

$\overrightarrow{AC}$ bisects $\angle$ BAM and intersects circle M at C

If D is the midpoint of $\overline{AB}$

Prove that : $\overline{DM} \perp \overline{CM}$



(El-Beheira 19 , El-Gharbia 17 , Souhag 14)

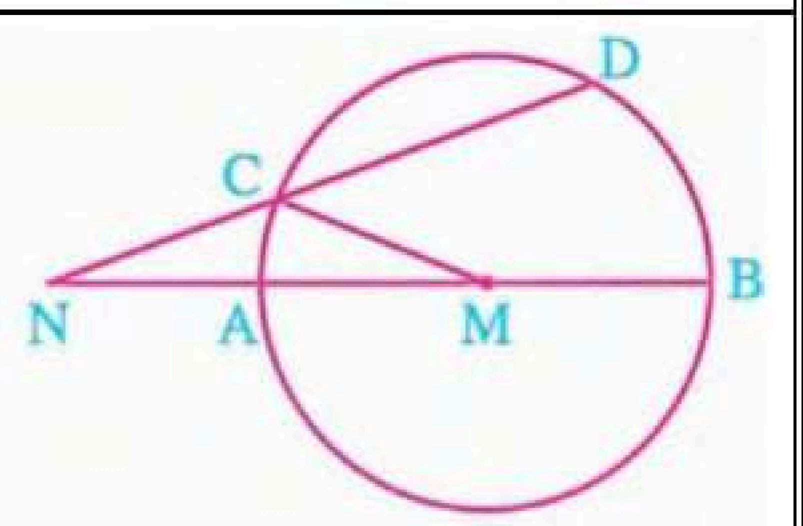
11

In the opposite figure :

$\overline{AB}$ is a diameter in circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NC > NA



« El-Beheira 18 »

12

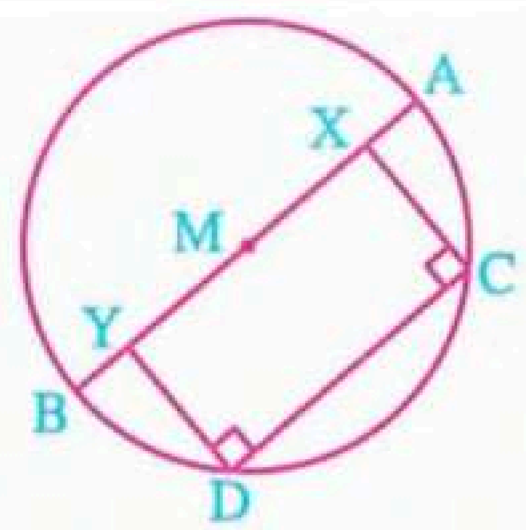
In the opposite figure :

$\overline{AB}$ is a diameter of the circle M ,

$\overline{CD}$ is a chord of it , $\overline{XC} \perp \overline{CD}$

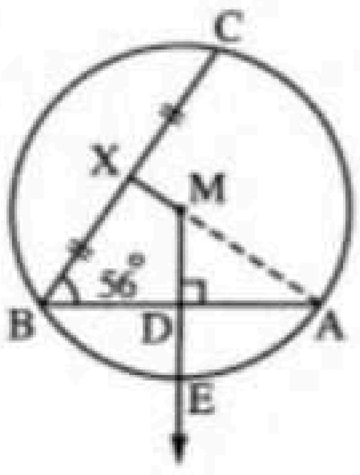
and $\overline{YD} \perp \overline{CD}$

Prove that : AX = BY

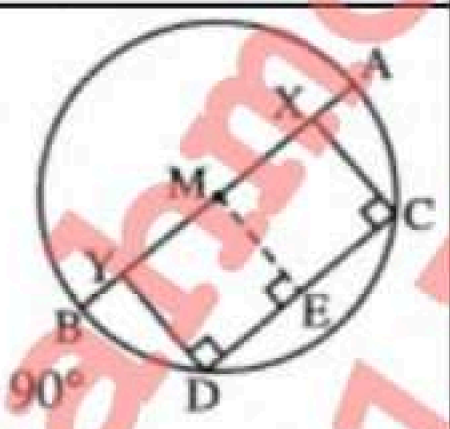


(Sharkia 09)

Solutions

A	Essay Problems
1	$\because$ MA = MD = r $\therefore$ Δ AMD is an isosceles triangle. $\therefore$ m ($\angle$ DAM) = m ($\angle$ ADM) = 25° $\therefore$ m ($\angle$ DAC) = 25° + 40° = 65° $\therefore$ In Δ ADC : m ($\angle$ ACD) = 180° – (25° + 65°) = 90° $\therefore$ $\overline{DC} \perp \overline{AB}$ $\because$ M $\in \overline{DC}$ $\therefore$ C is the midpoint of $\overline{AB}$ (Q.E.D)
2	 $\because$ X is the midpoint of $\overline{CB}$ $\therefore$ $\overline{MX} \perp \overline{BC}$ $\therefore$ m ($\angle$ DMX) = 360° – (90° + 90° + 56°) = 124° (First req.) , $\because$ $\overline{MD} \perp \overline{AB}$ $\therefore$ D is the midpoint of $\overline{AB}$ $\therefore$ AD = 4 cm. In Δ ADM : $\because$ m ($\angle$ ADM) = 90° , AM = r = 5 cm. $\therefore$ MD = $\sqrt{(AM)^2 - (AD)^2} = \sqrt{25 - 16}$ $= \sqrt{9} = 3$ cm. $\therefore$ DE = 5 – 3 = 2 cm. (Second req.)
3	$\because$ D is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MD} \perp \overline{AB}$ $\therefore$ m ($\angle$ BDM) = 90° similarly m ($\angle$ MEA) = 90° $\therefore$ From Δ AFE : m ($\angle$ DFM) = 45° and from Δ DFM : m ($\angle$ DMF) = 45° $\therefore$ Δ DFM is an isosceles triangle. (Q.E.D)
4	$\because$ C is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MD} \perp \overline{AB}$ In Δ ACM : $\because$ m ($\angle$ ACM) = 90° $\therefore$ $(MC)^2 = (AM)^2 - (AC)^2$ (Pythagoras' theorem) $\therefore$ $(MC)^2 = (13)^2 - (12)^2 = 25$ $\therefore$ MC = 5 cm. $\therefore$ CD = MD – MC = 13 – 5 = 8 cm. $\therefore$ The area of Δ ADB = $\frac{1}{2} \times 24 \times 8 = 96$ cm^2. (The req.)

5	$\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$ $\therefore$ m ($\angle$ AXY) = 90° , $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{XY}$ is a transversal $\therefore$ m ($\angle$ XYD) = m ($\angle$ AXY) = 90° (alternate angles) $\therefore$ $\overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$ (Q.E.D)
6	$\because$ D is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MD} \perp \overline{AB}$ $\because$ E is the midpoint of $\overline{AC}$ $\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ DME) = 360° – (120° + 90° + 90°) = 60° $\therefore$ m ($\angle$ XMY) = m ($\angle$ DME) = 60° (V.O.A) $\because$ MX = MY = r $\therefore$ Δ XYM is an equilateral triangle. (Q.E.D)
7	$\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$ $\therefore$ m ($\angle$ AXY) = 90° – 30° = 60° , $\because$ AB = AC $\therefore$ $\frac{1}{2}$ AB = $\frac{1}{2}$ AC $\therefore$ AX = AY , $\because$ m ($\angle$ AXY) = 60° $\therefore$ Δ AXY is an equilateral triangle. (Q.E.D.)
8	In the great circle : $\because$ $\overline{ME} \perp \overline{AB}$ $\therefore$ E is the midpoint of $\overline{AB}$ $\therefore$ AE = EB (1) In the small circle : $\because$ $\overline{ME} \perp \overline{CD}$ $\therefore$ E is the midpoint of $\overline{CD}$ $\therefore$ CE = ED (2) Subtracting (2) from (1) : $\therefore$ AE – CE = EB – ED $\therefore$ AC = BD (Q.E.D)
9	$\because$ $\overline{MD} \perp \overline{BC}$ $\therefore$ D is the midpoint of $\overline{BC}$ $\because$ $\overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$ $\therefore$ In Δ ABC : $\because$ D and E are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. $\therefore$ $\overline{ED}$ // $\overline{AB}$ (Q.E.D 1)

	$\because$ D is the midpoint of $\overline{BC}$ $\therefore DC = \frac{1}{2} BC$ (1) $\because$ E is the midpoint of $\overline{AC}$ $\therefore EC = \frac{1}{2} AC$ (2) $\because$ D and E are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. $\therefore DE = \frac{1}{2} AB$ (3) Adding (1) , (2) and (3) : $\therefore$ The perimeter of Δ CDE $= \frac{1}{2}$ the perimeter of Δ ABC (Q.E.D. 2)
10	In Δ AMC : $\because AM = MC = r \quad \therefore m(\angle MAC) = m(\angle ACM)$ $\because m(\angle BAC) = m(\angle MAC)$ $\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles $\therefore \overline{AB} \parallel \overline{CM}$ $\because$ D is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MD} \perp \overline{AB}$ $\because \overline{AB} \parallel \overline{CM} \quad \therefore \overline{DM} \perp \overline{CM}$ (Q.E.D)
11	In Δ MNC : $\because NC + MC > NM$ (triangle inequality) $, \because MA = MC = r \quad , \quad NM = AN + MA$ $\therefore NC + MC > AN + MA$ $\therefore NC > AN$ (Q.E.D.)
12	**Construction :** Draw $\overrightarrow{ME} \perp \overleftrightarrow{CD}$ to cut it at E  **Proof :** $\because \overline{ME} \perp \overline{CD}$ $\therefore$ E is the midpoint of $\overline{CD}$ $, m(\angle XCE) = m(\angle MED) = 90°$ but they are corresponding angles $\therefore \overline{XC} \parallel \overline{ME}$ similarly $\overline{ME} \parallel \overline{YD}$ $\therefore \overline{XC} \parallel \overline{ME} \parallel \overline{YD}$ $\because \overleftrightarrow{XY}$ and $\overleftrightarrow{CD}$ are two transversals to them $, CE = ED \quad \therefore XM = MY$ $\because AM = BM = r \quad \therefore AM - XM = BM - MY$ $\therefore AX = BY$ (Q.E.D.)

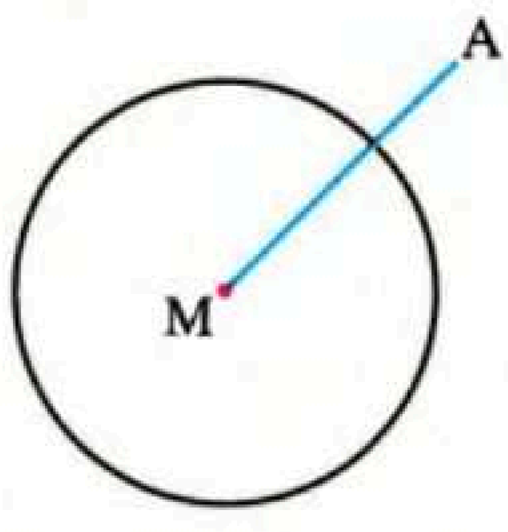
Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [2] : Positions Of A Point and A Straight Line With Respect To A Circle

First Position of a point with respect to a given circle

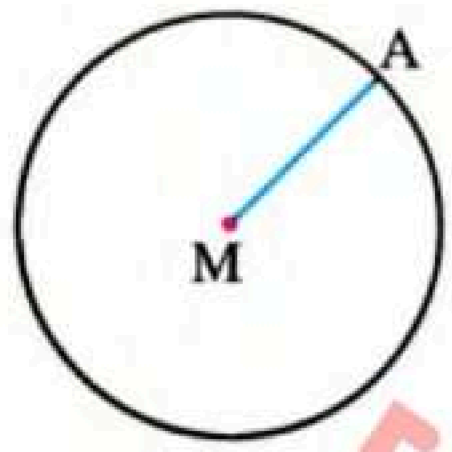
If M is a circle of radius length r and A is a point in its plane , then :

1 A is outside the circle M



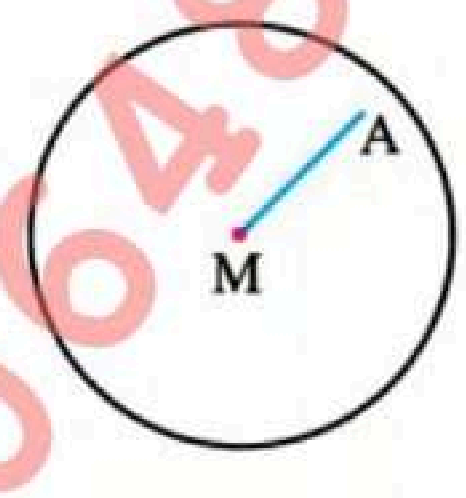
If MA > r

2 A is on the circle M



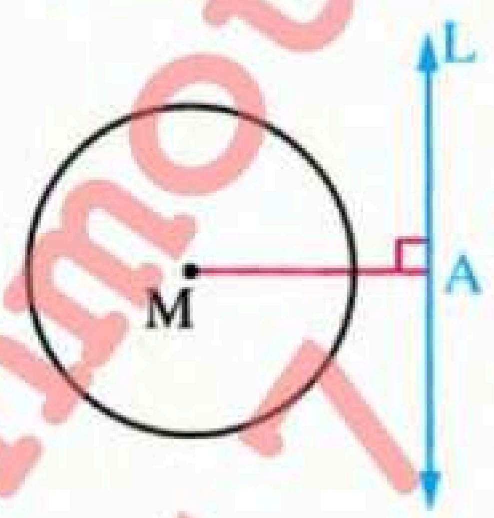
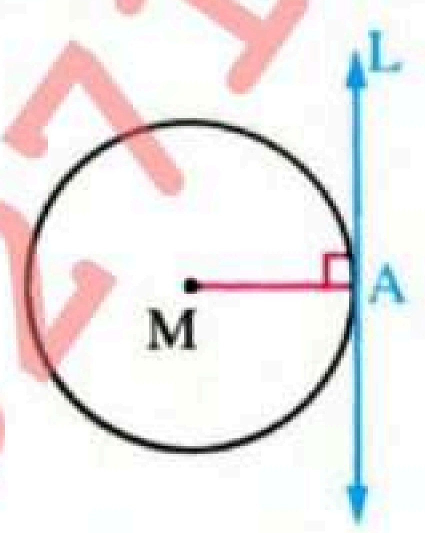
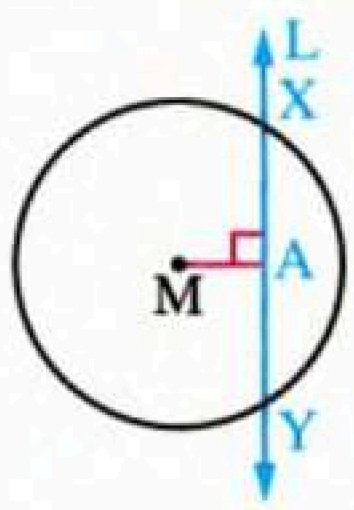
If MA = r

3 A is inside the circle M



If MA < r

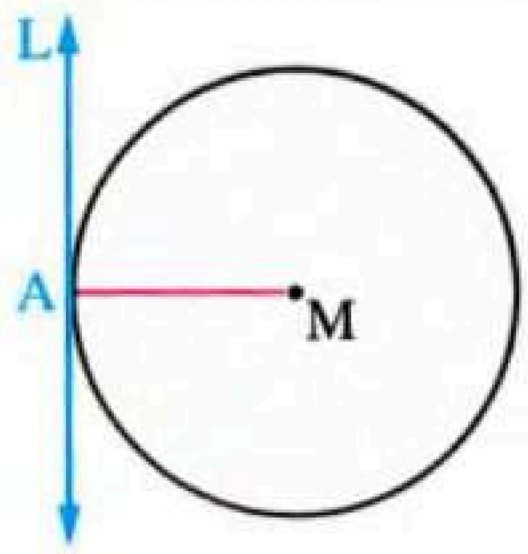
Second Position of a straight line with respect to a circle

If	Then	The figure	Note that
1 MA > r	The straight line L lies outside the circle M		• L ∩ the circle M = ∅ • L ∩ the surface of the circle M = ∅
2 MA = r	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		• L ∩ the circle M = {A} • L ∩ the surface of the circle M = {A}
3 MA < r	The straight line L is a secant to the circle M		• L ∩ the circle M = {X , Y} • L ∩ the surface of the circle M = $\overline{XY}$ • $\overline{XY}$ is called the chord of intersection

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

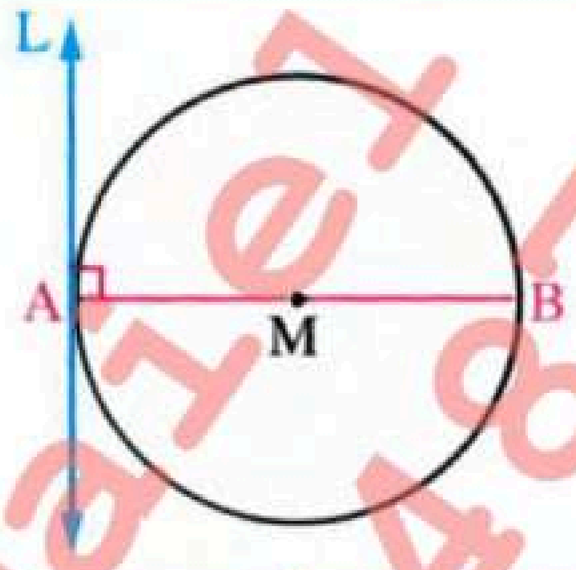
Two important facts

1 **The tangent to a circle is perpendicular to the radius drawn from the point of tangency.**



i.e. if the straight line L is a tangent to the circle M at the point A ,
then $\overline{MA} \perp L$

2 **The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.**



i.e. if $\overline{AB}$ is a diameter of the circle M and the straight line $L \perp \overline{AB}$ at the point A ,
then L is a tangent to the circle M at the point A

Exercises

[A] Essay problems : -

1

In the opposite figure :

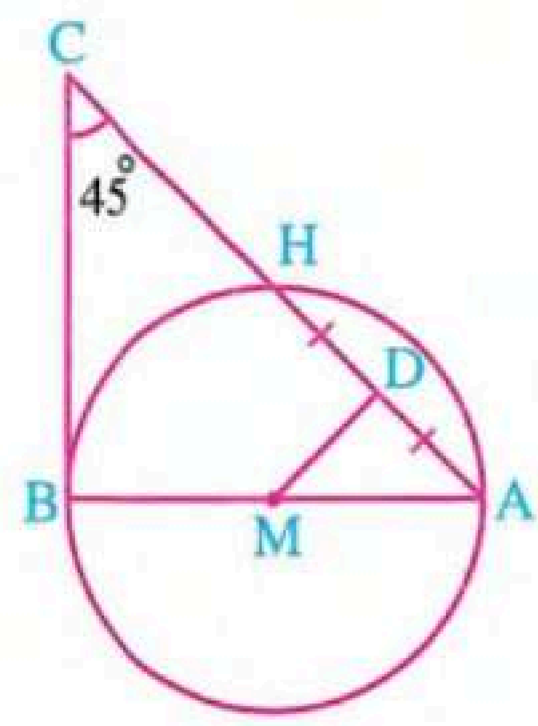
$\overrightarrow{BC}$ is a tangent at B

, m (∠ C) = 45°

, D is the midpoint of $\overline{AH}$

Prove that : DA = DM

(Aswan 11)



2

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

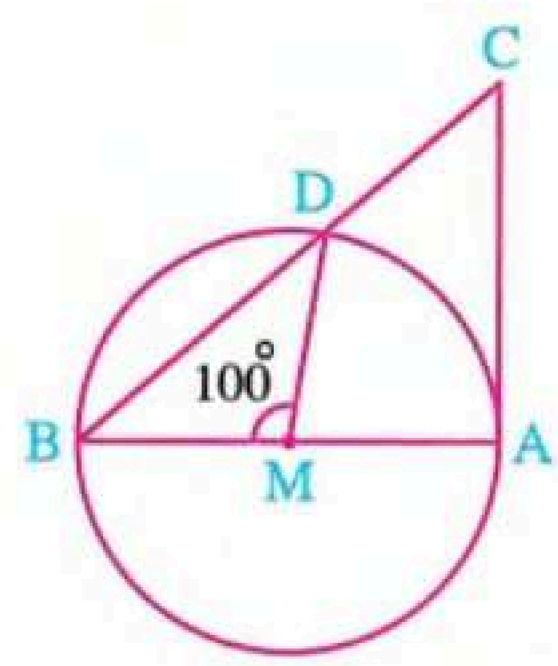
$\overrightarrow{AC}$ is a tangent to the circle at A ,

m (∠ DMB) = 100°

Find by proof :

1 m (∠ ACB)

2 m (∠ CDM)



(El-Menia 11) « 50° , 140° »

3

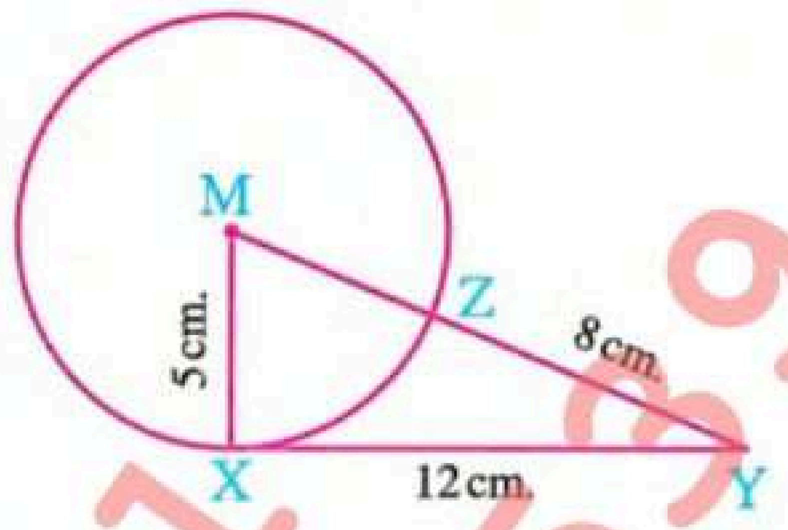
In the opposite figure :

M is a circle with radius length 5 cm. ,

XY = 12 cm. , $\overline{MY} \cap$ circle M = $\{Z\}$

and ZY = 8 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



(Matrouh 17 , South Sinai 16 , Qena 15 , El-Beheira 14)

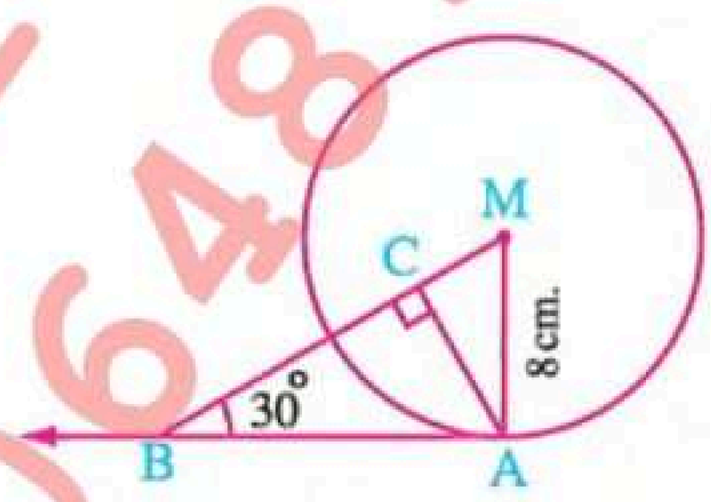
4

In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M at A ,

MA = 8 cm. , m (∠ ABM) = 30° and $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$ and $\overline{AC}$



(Giza 19 , Matrouh 18 , New Valley 18 , El-Monofia 14) « $8\sqrt{3}$ cm. , $4\sqrt{3}$ cm. »

5

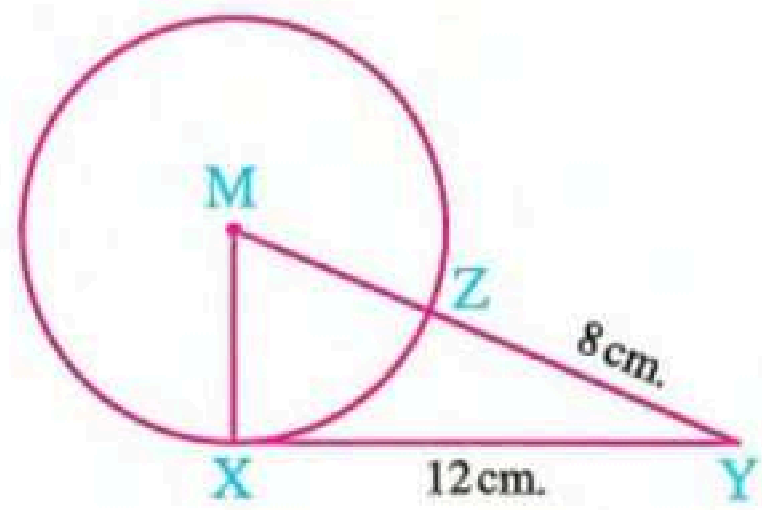
In the opposite figure :

M is a circle , $\overleftrightarrow{XY}$ is a tangent to the circle at X

, $\overline{MY} \cap$ the circle M = $\{Z\}$,

XY = 12 cm. , YZ = 8 cm.

Find : The radius length of the circle.



(El-Menia 13) « 5 cm. »

6

In the opposite figure :

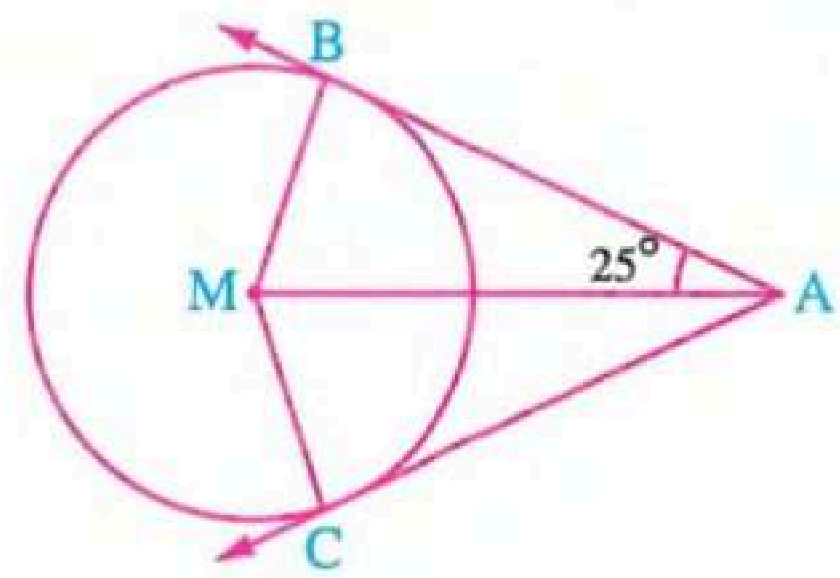
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, touch it at B , C respectively

and m (∠ BAM) = 25°

1 **Prove that :** $\overrightarrow{MA}$ bisects ∠ BMC

2 **Find :** m (∠ BMC)



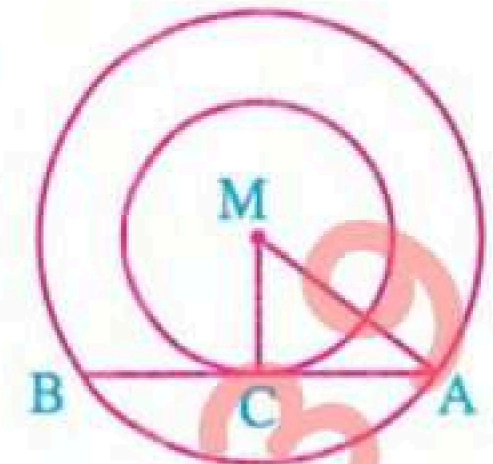
(Port Said 17) « 130° »

7

In the opposite figure :

$\overline{AB}$ is a chord of the great circle and touches the small circle at C , AB = 8 cm. and the radius length of the great circle = 5 cm.

Find : The radius length of the small circle.



(Souhag 09) « 3 cm. »

8

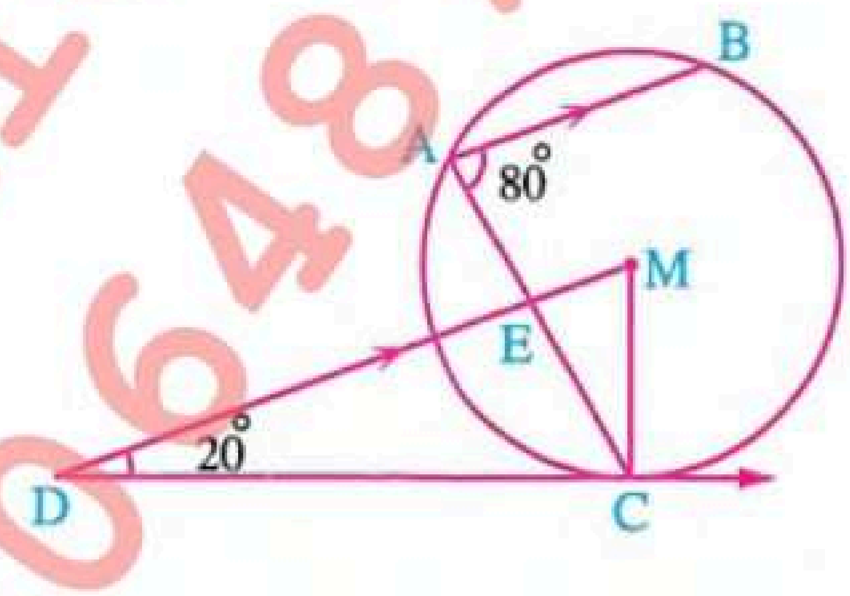
In the opposite figure :

$\overleftrightarrow{DC}$ touches the circle M at C , $\overline{AB}$ // $\overline{MD}$,

m (∠ BAC) = 80° , m (∠ MDC) = 20°

and $\overline{AC} \cap \overline{MD} = \{E\}$

Find : m (∠ ECM)



(Beni Suef 05) « 30° »

9

$\overline{AB}$ is a diameter in a circle of area 36 π cm^2 , $\overleftrightarrow{BC}$ is drawn a tangent to the circle at B , if m (∠ ACB) = 60° , then calculate the area of Δ ABC *(El-Dakahlia 14)* « $24\sqrt{3}$ cm^2 »

10

Prove that : The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located in circle whose centre is the point M (– 1 , 2) , then find the circumference of the circle.

(El-Beheira 11) «10 π length units »

11

If $\overline{CD}$ is a diameter of circle M where M (1 , 1) , D (3 , –2)

Find : The equation of the tangent to M at C *(El-Dakahlia 11)* « $y = \frac{2}{3} x + 4\frac{2}{3}$ »

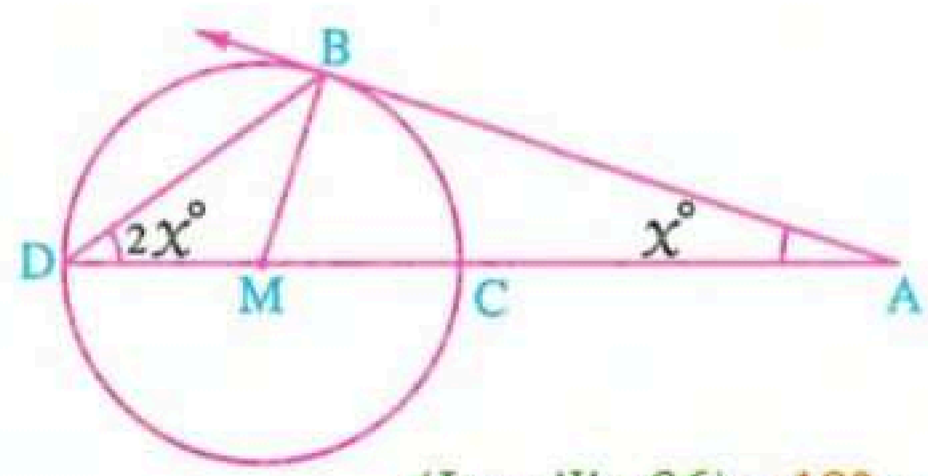
12

In the opposite figure :

$\overrightarrow{AB}$ touches the circle M at B , $\overline{CD}$ is a diameter of it ,

m (∠ BAM) = x° and m (∠ MDB) = 2 x°

Find : The value of x in degrees.



(Ismailia 06) « 18° »

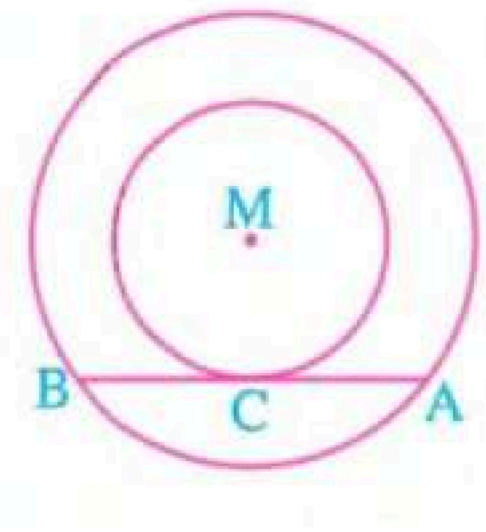
13

In the opposite figure :

Two circles are concentric at M

, $\overline{AB}$ is a chord in the greater circle and touches the smaller circle at C , if AB = 14 cm.

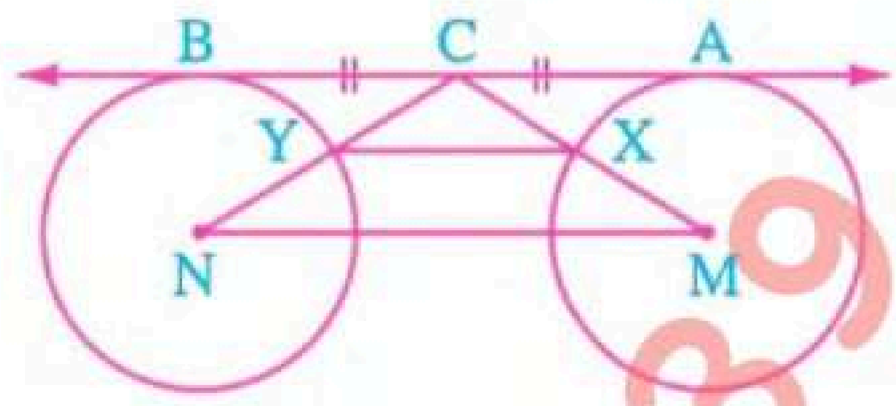
Find : The area of the part included between the two circles.



(El-Dakahlia 19) « 49 π cm^2 »

14

In the opposite figure :



M and N are two congruent circles ,

$\overleftrightarrow{AB}$ is a common tangent to them ,

C is the midpoint of $\overline{AB}$,

the circle $M \cap \overline{MC} = \{X\}$, the circle $N \cap \overline{NC} = \{Y\}$

Prove that : 1 $\overline{AB} \parallel \overline{MN}$

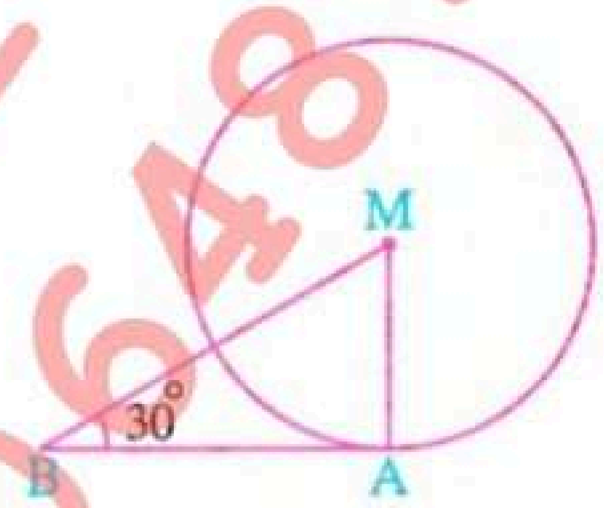
2 Δ CMN is an isosceles triangle.

3 $\overline{XY} \parallel \overline{MN}$

(El-Kalyoubia 04)

[B] Choose the correct : -

1	If a straight line L is a tangent to the circle M whose diameter length is 8 cm. , then L is at a distance of .... cm. from its centre. *(Souhag 19 , El-Kalyoubia 18)* (a) 3 (b) 4 (c) 6 (d) 8
2	A circle M is of radius length 5 cm. , A is a point outside the circle , then MA equals .... cm. *(Gharbia 03)* (a) 3 (b) 5 (c) 8 (d) 4
3	If the diameter length of a circle is 8 cm. and the straight line L is at distance of 3 cm. from its centre , then the straight line L is .... *(Damietta 16 , El-Menia 14)* (a) a tangent to the circle. (b) a secant to the circle. (c) outside the circle. (d) an axis of symmetry of the circle.
4	If M is a circle its diameter length = 14 cm. , MA = $(2x + 3)$ cm. where A is a point on the circle , then x = ... *(El-Kalyoubia 17 , El-Sharkia 15)* (a) 5 (b) 3 (c) 2 (d) 1
5	📖 $\overline{AB}$ is a diameter in a circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle , then $\overleftrightarrow{AC}$ .... $\overleftrightarrow{BD}$ *(Alexandria 13)* (a) intersects (b) is perpendicular to (c) is parallel to (d) is coincident to
6	📖 A circle is of a circumference 6π cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is .... *(Red Sea 19 , Red Sea 17 , El-Monofia 15)* (a) a tangent to the circle. (b) a secant. (c) outside the circle. (d) a diameter of the circle.
7	If the area of the circle M is 16π cm^2 , A is a point in its plane where MA = 8 cm. , then A lies .... the circle M *(Qena 17 , El-Sharkia 09)* (a) inside (b) outside (c) on (d) at the centre of
8	M is a circle with diameter of length 8 cm. If the straight line L is outside the circle , then the distance between the centre of the circle and the straight line L $\in$ .... (a) $]4 , \infty[$ (b) $[0 , 4[$ (c) $]0 , 4[$ (d) $[0 , 8]$ *(Kafr El-Sheikh 14)*

9	A circle with diameter length $(2X+5)$ cm. , the straight line L is at a distance $(X+2)$ cm. from its centre , then the straight line L is .... *(Port Said 17)* (a) a secant to the circle at the two points. (b) outside the circle. (c) a tangent to the circle. (d) an axis of symmetry of the circle.
10	$\overrightarrow{AB}$ is a tangent to the circle M , m ($\angle$ B) = 30° , AM = 6 cm. , then MB = ... cm. *(Red Sea 18)* (a) 3 (b) 6 (c) 9 (d) 12 

Solutions

A — Essay Problems

1

$\because \overrightarrow{BC}$ is a tangent to the circle M at B

$\therefore \overline{BC} \perp \overline{MB}$

In Δ ABC : m ($\angle$ A) = 180° – (45° + 90°) = 45°

, $\because$ D is the midpoint of $\overline{AH}$ $\therefore \overline{MD} \perp \overline{AH}$

In Δ ADM :

m ($\angle$ DMA) = 180° – (45° + 90°) = 45°

$\therefore$ m ($\angle$ DAM) = m ($\angle$ DMA)

$\therefore$ DA = DM (Q.E.D.)

2

In Δ MDB :

$\because$ MD = MB = r

$\therefore$ m ($\angle$ MBD) = m ($\angle$ MDB) = $\frac{180° - 100°}{2}$ = 40°

, $\because \overleftrightarrow{AC}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overline{AC}$

In Δ ABC :

m ($\angle$ C) = 180° – (90° + 40°) = 50° (First req.)

, m ($\angle$ CDM) = 180° – 40° = 140° (Second req.)

3

$\because$ MZ = r = 5 cm. $\therefore$ MY = 13 cm.

, $\because (MY)^2 = 169$, $(MX)^2 = 25$

, $(XY)^2 = 144$

$\therefore (MX)^2 + (XY)^2 = (MY)^2$

$\therefore$ m ($\angle$ MXY) = 90° $\therefore \overline{XY} \perp \overline{MX}$

$\therefore \overleftrightarrow{XY}$ is a tangent to the circle M at X (Q.E.D.)

4

$\because \overrightarrow{AB}$ is a tangent to the circle M at A

$\therefore$ m ($\angle$ MAB) = 90°

$\therefore \tan(\angle B) = \frac{MA}{AB}$ $\therefore \tan 30° = \frac{8}{AB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{8}{AB}$ $\therefore AB = 8\sqrt{3}$ cm.

In Δ ABC which is right-angled at C

$\because$ m ($\angle$ ABC) = 30°

$\therefore AC = \frac{1}{2} AB = \frac{1}{2} \times 8\sqrt{3}$

$= 4\sqrt{3}$ cm. (Second req.)

5

$\because \overleftrightarrow{XY}$ is a tangent to the circle at X

$\therefore \overline{MX} \perp \overline{XY}$ $\therefore$ m ($\angle$ MXY) = 90°

$\therefore$ In Δ MXY : $(MY)^2 = (MX)^2 + (XY)^2$

$\therefore (MZ + 8)^2 = (MX)^2 + 144$

, $\because$ MZ = MX = r $\therefore (r + 8)^2 = r^2 + 144$

$\therefore r^2 + 16r + 64 = r^2 + 144$ $\therefore 16r = 80$

$\therefore r = \frac{80}{16} = 5$ cm. (The req.)

6

$\because \overrightarrow{AB}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overrightarrow{AB}$ $\therefore$ m ($\angle$ ABM) = 90°

, $\because \overrightarrow{AC}$ is a tangent to the circle M at C

$\therefore \overline{MC} \perp \overrightarrow{AC}$ $\therefore$ m ($\angle$ ACM) = 90°

$\therefore$ In $\Delta\Delta$ ABM , ACM which are right-angled

$\begin{cases} MB = MC = r \\ \overline{AM} \text{ is a common hypotenuse} \end{cases}$

$\therefore \Delta ABM \equiv \Delta ACM$

$\therefore$ m ($\angle$ AMB) = m ($\angle$ AMC)

$\therefore \overrightarrow{MA}$ bisects $\angle$ BMC (First req.)

From Δ ABM : m ($\angle$ AMB) = 180° – (90° + 25°)

= 65°

$\therefore$ m ($\angle$ BMC) = 2 × 65° = 130° (Second req.)

7

In the small circle :

$\because \overleftrightarrow{AB}$ is a tangent atc $\therefore \overline{MC} \perp \overleftrightarrow{AB}$

In the great circle : $\because \overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$ $\therefore$ AC = 4 cm.

, $\because$ AM = 5 cm.

$\therefore$ In Δ ACM which is right-angled at C

$MC = \sqrt{(AM)^2 - (AC)^2} = \sqrt{25 - 16} = 3$ cm. (The req.)

8

$\because \overrightarrow{DC}$ is a tangent to the circle M at C

$\therefore \overline{MC} \perp \overrightarrow{DC}$ $\therefore$ m ($\angle$ MCD) = 90°

$\therefore$ In Δ DMC : m ($\angle$ DMC) = 180° – (90° + 20°) = 70°

, $\because \overline{AB} \parallel \overline{MD}$, $\overleftrightarrow{AE}$ is a transversal to them

$\therefore$ m ($\angle$ MEC) = m ($\angle$ BAE) = 80°

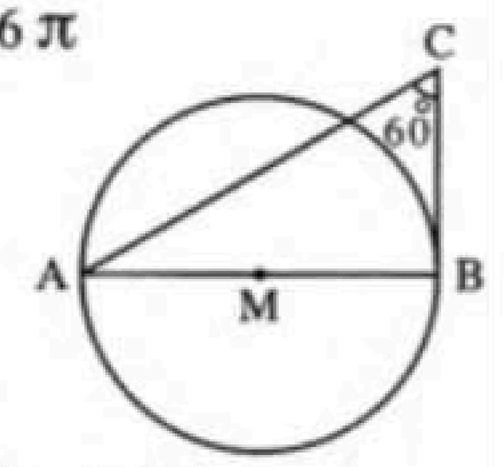
(corresponding angles)

$\therefore$ In Δ MEC :

m ($\angle$ ECM) = 180° – (70° + 80°) = 30° (The req.)

9

$\because$ The area of the circle $= 36\pi$



$\therefore r^2\pi = 36\pi$

$\therefore r^2 = 36$

$\therefore r = 6$ cm.

$\therefore AB = 12$ cm.

, $\because \overleftrightarrow{BC}$ is a tangent to the circle M at B

$\therefore \overline{BC} \perp \overline{AB}$

In ΔABC : $\tan(\angle C) = \frac{AB}{BC}$ $\quad \therefore \tan 60° = \frac{12}{BC}$

$\therefore BC = \frac{12}{\tan(60°)} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ cm.

$\therefore$ The area of ΔABC

$= \frac{1}{2} AB \times BC = \frac{1}{2} \times 12 \times 4\sqrt{3}$

$= 24\sqrt{3}$ cm^2. (The req.)

10

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25}$

$= 5$ length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25}$

$= 5$ length units

, $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}$

$= 5$ length units

$\therefore MA = MB = MC$

$\therefore$ The points A , B and C lie on the circle M (Q.E.D.1)

, its circumference $= 10\pi$ length units. (Q.E.D.2)

11

$\because \overline{CD}$ is a diameter in the circle M

$\therefore$ M is the midpoint of $\overline{CD}$

Let C (x , y) $\quad \therefore (1 , 1) = \left(\frac{x+3}{2} , \frac{y-2}{2}\right)$

$\therefore \frac{x+3}{2} = 1$ $\quad \therefore x + 3 = 2$ $\quad \therefore x = -1$

, $\frac{y-2}{2} = 1$ $\quad \therefore y - 2 = 2$ $\quad \therefore y = 4$

$\therefore C = (-1 , 4)$

, $\because$ the slope of $\overleftrightarrow{CD} = \frac{-2-4}{3+1} = \frac{-6}{4} = -\frac{3}{2}$

$\therefore$ The slope of the perpendicular straight line to $\overleftrightarrow{CD} = \frac{2}{3}$

, $\because$ the tangent to the circle M at C is perpendicular to $\overline{CD}$

$\therefore$ The slope of the tangent to the circle at C $= \frac{2}{3}$

$\therefore$ The equation of the tangent is : $y = \frac{2}{3}x + c$

, $\because$ the tangent passes through the point C $(-1 , 4)$

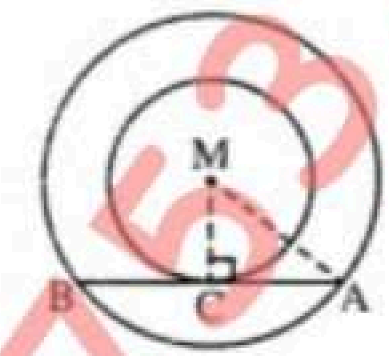
$\therefore 4 = \frac{2}{3} \times -1 + c$ $\quad \therefore c = 4\frac{2}{3}$

$\therefore$ The equation is : $y = \frac{2}{3}x + 4\frac{2}{3}$ (The req.)

12

$\because \overline{AB}$ touches the small circle at C

$\therefore \overline{MC} \perp \overline{AB}$



, $\because \overline{AB}$ is a chord of the great circle , $\overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{14}{2} = 7$ cm.

$\because \Delta AMC$ is right angled at C

$\therefore (AC)^2 = (MA)^2 - (MC)^2$

$\therefore (7)^2 = (MA)^2 - (MC)^2$

$\therefore (MA)^2 - (MC)^2 = 49$

$\therefore$ The area of the included part between the two circles = the area of the greater circle – the area of the smaller circle $= \pi(MA)^2 - \pi(MC)^2$

$= \pi[(MA)^2 - (MC)^2] = 49\pi$ cm^2. (The req.)

13

$\because \overleftrightarrow{AB}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overleftrightarrow{AB}$ $\quad \therefore m(\angle ABM) = 90°$

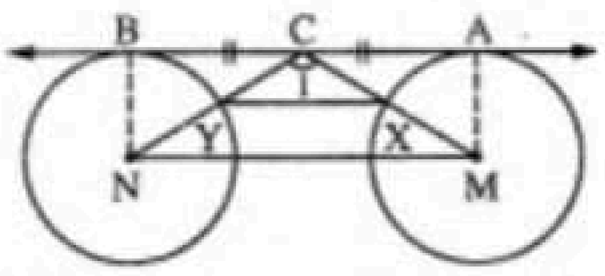
$\because MB = MD$ (lengths of two radii)

$\therefore m(\angle MBD) = m(\angle MDB) = 2x°$

In ΔABD : $m(\angle A) + m(\angle ABD) + m(\angle D) = 180°$

$\therefore x° + 90° + 2x° + 2x° = 180°$

$\therefore 5x° = 90°$ $\quad \therefore x = 18°$ (The req.)

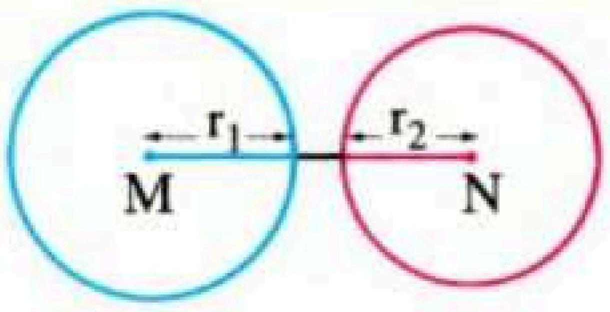
14	**Construction :** We draw $\overline{MA}$ and $\overline{NB}$
	Proof : ∵ $\overleftrightarrow{AB}$ is a tangent to the circle M at A
	∴ $\overline{MA} \perp \overleftrightarrow{AB}$
	
	similarly $\overline{NB} \perp \overleftrightarrow{AB}$
	∴ $\overline{MA} // \overline{BN}$
	, ∵ MA = NB (two radii of two congruent circles)
	∴ AMNB is a parallelogram
	∴ $\overline{AB} // \overline{MN}$ (Q.E.D. 1)
	In ΔΔ AMC , BNC :
	AC = BC (given)
	MA = BN (given)
	m (∠ MAC) = m (∠ NBC) = 90° (proved)
	∴ Δ AMC ≡ Δ BNC ∴ MC = NC (1)
	∴ Δ CMN is an isosceles triangle (Q.E.D. 2)
	∵ MX = NY (2)
	∴ Subtracting (2) from (1) :
	∴ MC − MX = NC − NY ∴ CX = CY
	∴ From the isosceles triangle XCY
	∴ $m(\angle CXY) = \frac{180^\circ - m(\angle 1)}{2}$ (3)
	and from the isosceles triangle MNC
	∴ $m(\angle CMN) = \frac{180^\circ - m(\angle 1)}{2}$ (4)
	From (3) and (4) : m (∠ CXY) = m (∠ CMN)
	and they are corresponding angles
	∴ $\overline{XY} // \overline{MN}$ (Q.E.D. 3)

B	Choose
1	B
2	C
3	B
4	C
5	C
6	A
7	B
8	A
9	A
10	D

Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [3] : Positions Of A Circle With Respect To Another Circle

If $MN > r_1 + r_2$

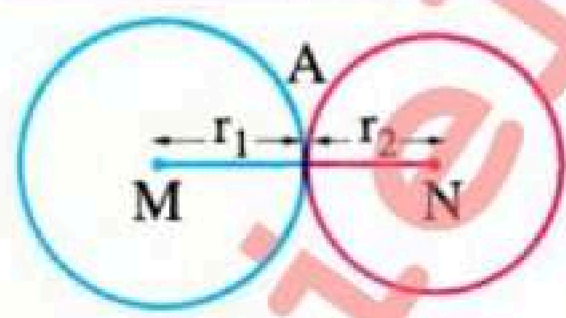


Then the two circles are : Distant

Notice that :

- The circle M $\cap$ the circle N = $\varnothing$
- The surface of circle M $\cap$ the surface of circle N = $\varnothing$

If $MN = r_1 + r_2$

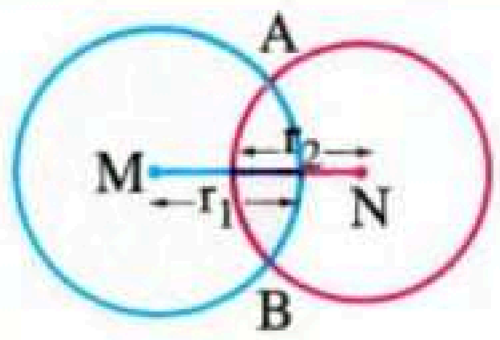


Then the two circles are :
Touching externally

Notice that :

- The circle M $\cap$ the circle N = $\{A\}$
- The surface of circle M $\cap$ the surface of circle N = $\{A\}$

If $r_1 - r_2 < MN < r_1 + r_2$

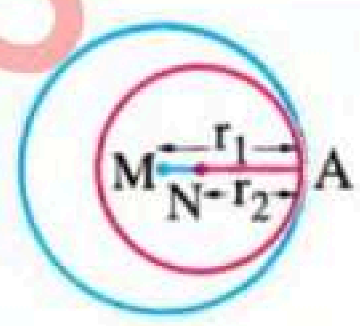


Then the two circles are :
Intersecting

Notice that :

- The circle M $\cap$ the circle N = $\{A , B\}$
- The surface of circle M $\cap$ the surface of circle N = the surface of the shaded part.

If $MN = r_1 - r_2$

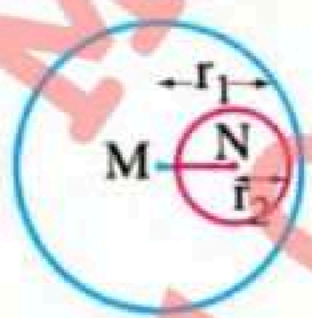


Then the two circles are :
Touching internally

Notice that :

- The circle M $\cap$ the circle N = $\{A\}$
- The surface of circle M $\cap$ the surface of circle N = the surface of circle N

If $MN < r_1 - r_2$



Then the two circles are :
One inside the other
(the circle N is inside the circle M)

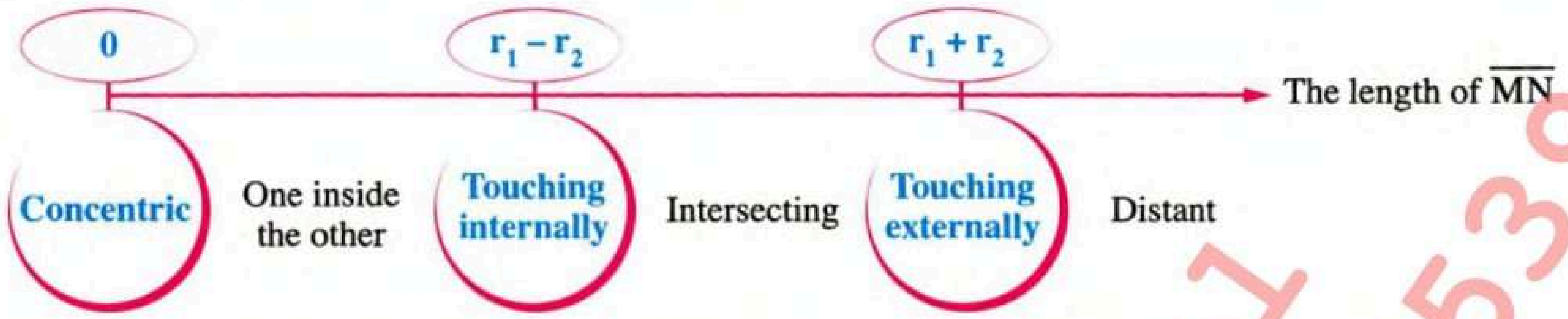
If MN = zero

Then the two circles are :
Concentric

Notice in the two cases that :

- The circle M $\cap$ the circle N = $\varnothing$
- The surface of circle M $\cap$ the surface of circle N = the surface of circle N

Summary



Remarks

From the previous summary , we notice that :

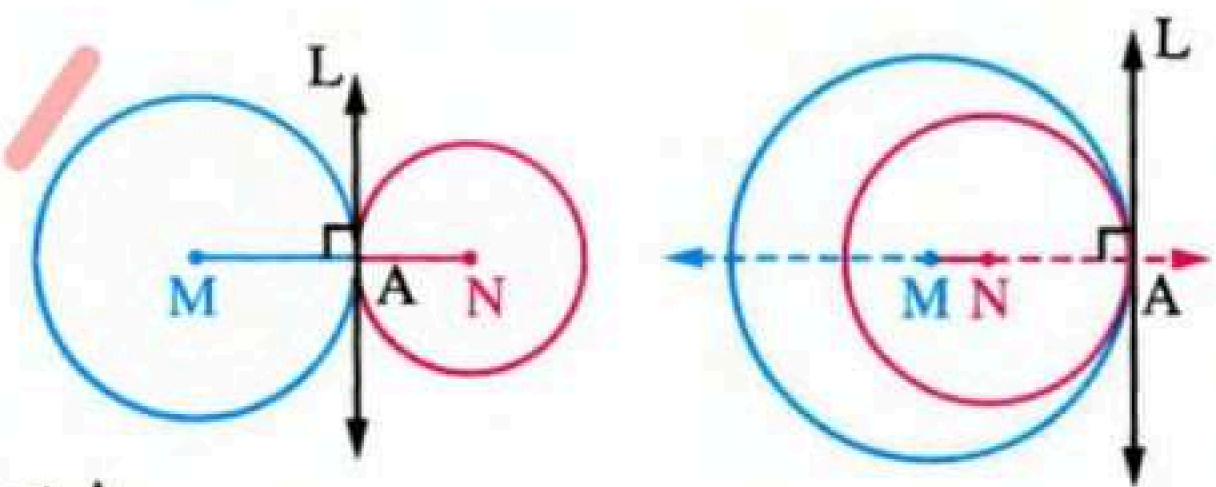
1. If M and N are two distant circles , then : $MN \in]\, r_1 + r_2 , \infty [$
2. If M and N are two intersecting circles , then : $MN \in]\, r_1 - r_2 , r_1 + r_2 [$
3. If M and N (one of them is inside the other) , then : $MN \in]\, 0 , r_1 - r_2 [$

Corollary 1

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

If the two circles M and N are touching at A (the point of tangency) , the straight line L is a common tangent to them at A , then $A \in \overleftrightarrow{MN}$ and $\overleftrightarrow{MN} \perp$ the straight line L



Corollary 2

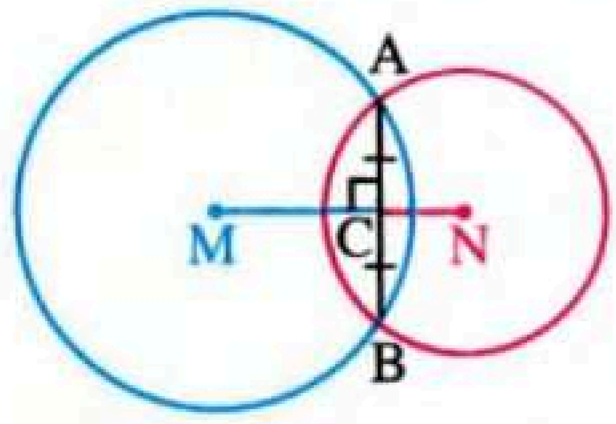
The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,

then $\overleftrightarrow{MN} \perp \overline{AB}$, $\overleftrightarrow{MN}$ bisects $\overline{AB}$ i.e. $AC = BC$

This mean that $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$



Exercises

[A] Essay problems : -

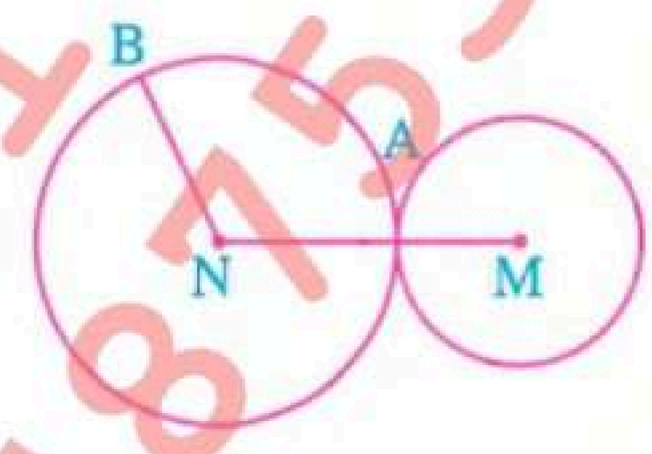
1 **In the opposite figure :**

M and N are two circles touching at A ,

the distance between their centres MN = 12 cm.

If NB = 7 cm.

Find : The length of $\overline{MA}$



(Kafr El-Sheikh 06) « 5 cm. »

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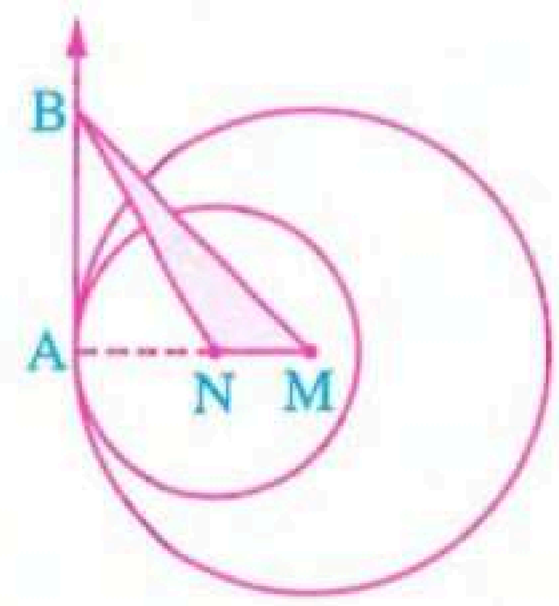
2 **In the opposite figure :**

M and N are two circles with radii lengths of 10 cm. and 6 cm. respectively and they are touching internally at A ,

$\overleftrightarrow{AB}$ is a common tangent for both.

If the area of Δ BMN = 24 cm^2.

Find : The length of $\overline{AB}$



(El-Kalyoubia 18 , Luxor 16 , Port Said 14) « 12 cm. »

---

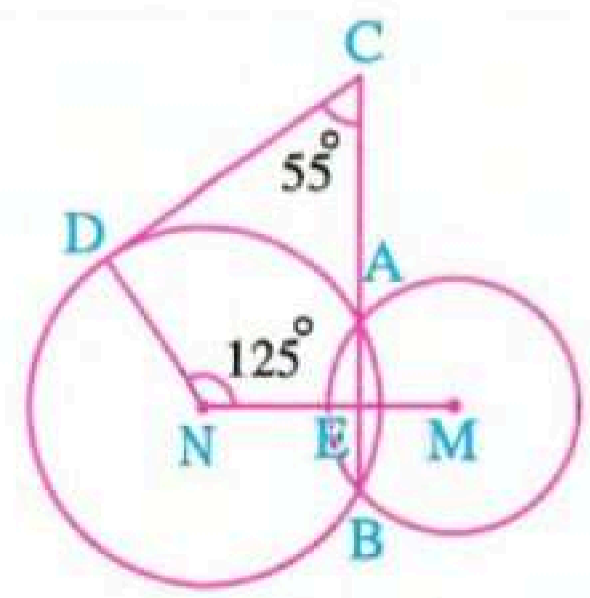
3 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

C $\in \overrightarrow{BA}$, D $\in$ the circle N ,

m ($\angle$ MND) = 125° and m ($\angle$ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D



(Red Sea 19 , Kafr El-Sheikh 17 , Souhag 15)

---

4 **In the opposite figure :**

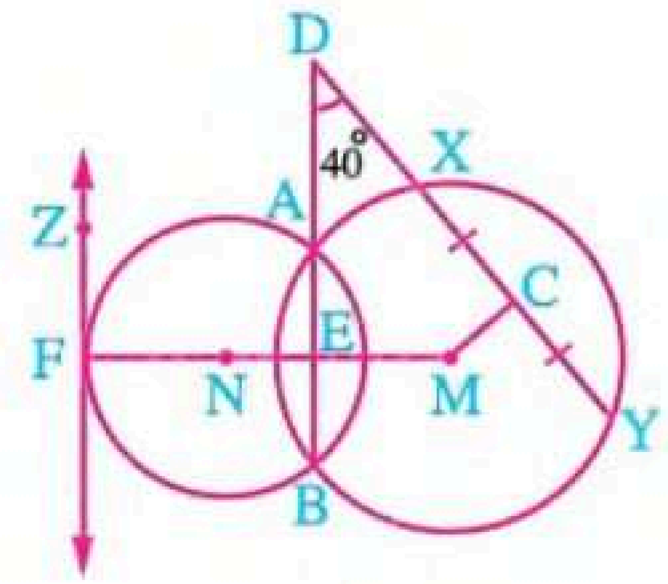
M and N are two intersecting circles at A and B ,

C is the midpoint of $\overline{XY}$, m ($\angle$ D) = 40° ,

$\overleftrightarrow{FZ}$ is a tangent to the circle N at F where $\overleftrightarrow{MN} \cap \overleftrightarrow{FZ} = \{F\}$

1 **Find :** m ($\angle$ CME) « 140° »

2 **Prove that :** $\overleftrightarrow{FZ}$ // $\overline{AB}$



(El-Fayoum 11)

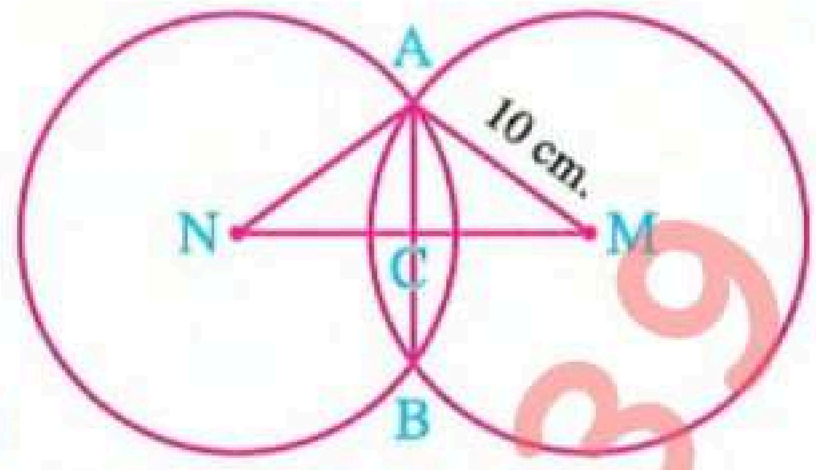
5

In the opposite figure :

Two congruent circles M and N are intersecting at A and B

If MA = 10 cm. , AB = 12 cm.

Find by proof : The length of $\overline{MN}$ *(El-Menia 17)* « 16 cm. »



6

M and N are two intersecting circles at A and B , MA = 12 cm , NA = 9 cm. and MN = 15 cm.

Find : The length of $\overline{AB}$ *(Port Said 11)* « 14.4 cm. »

7

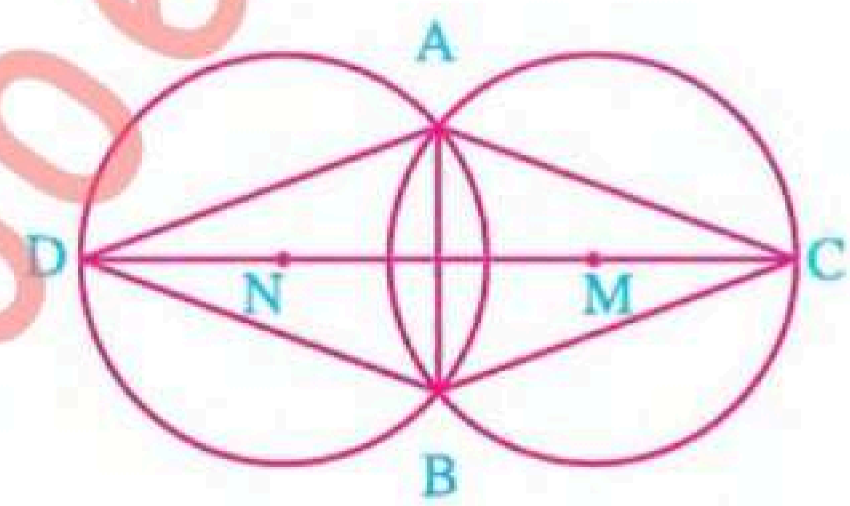
In the opposite figure :

M and N are two intersecting circles at A and B

where C is a point on the circle M ,

D is a point on the circle N , $C \in \overleftrightarrow{MN}$, $D \in \overleftrightarrow{MN}$

Prove that : m (∠ CAD) = m (∠ CBD) *(El-Sharkia 15)*



8

If M (3 , 5) and N (– 3 , – 7) are the two centres of two circles whose radii lengths are $4\sqrt{5}$ length units and $2\sqrt{5}$ length units respectively , A (– 1 , – 3)

Prove that : The two circles are touching at A showing the kind of tangency. *(Helwan 09)*

[B] Choose the correct : -

1	M and N are two circles touching internally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm. *(Beni Suef 17 , El-Gharbia 15)* (a) 8 (b) 6 (c) 4 (d) 2
2	M and N are two circles touching externally , if their radii lengths are 4 cm. and 2 cm. , then MN = ... cm. *(Cairo 15)* (a) zero (b) 2 (c) 6 (d) 7
3	M and N are two circles of radii lengths are 9 cm. and 4 cm. respectively , MN = 5 cm. , then the two circles are .... *(El-Dakahlia 17 , El-Gharbia 14)* (a) touching externally. (b) touching internally. (c) intersecting. (d) distant.
4	M and N are two circles , their radii lengths are 8 cm. and 3 cm. , if MN = 11 cm. , then the two circles M and N are .... *(El-Menia 13)* (a) distant. (b) concentric. (c) intersecting. (d) touching externally.
5	M and N are two circles , their radii lengths are 4 cm. and 3 cm. If MN = 9 cm. , then the two circles are .... *(Port Said 09)* (a) distant. (b) intersecting. (c) touching. (d) one is inside the other.
6	If the radii lengths of the two circles M and N are 6 cm. , 3 cm. , if MN = 2 cm. , then the two circles M , N are .... *(El-Dakahlia 18)* (a) intersecting. (b) one is inside the other. (c) touching externally. (d) distant.

7 If the radius length of the circle M = 3 cm. and the radius length of the circle N = 5 cm. , MN = 6 cm. , then the two circles M and N are .... *(El-Gharbia 08)*

(a) distant. (b) one is inside the other.
(c) intersecting. (d) touching externally.

8 🕮 M and N are two intersecting circles their radii lengths are 3 cm. and 5 cm. respectively , then MN $\in$ .... *(Alexandria 16 , Cairo 16 , Suez 11)*

(a) $]0 , 2[$ (b) $]2 , 8[$ (c) $]8 , \infty[$ (d) $]2 , \infty[$

9 Two circles M and N with radii lengths 8 cm. and 5 cm. respectively , are touching when MN $\in$ .... *(El-Dakahlia 16)*

(a) $]13 , 3[$ (b) $]3 , 13[$ (c) $\mathbb{R} - [3 , 13]$ (d) $\{13 , 3\}$

10 M and N are two intersecting circles at A and B , then the axis of symmetry of $\overline{AB}$ is .... *(El-Monofia 04)*

(a) $\overline{MN}$ (b) $\overrightarrow{NM}$ (c) $\overleftrightarrow{MN}$ (d) $\overrightarrow{MN}$

11 If the radius length of the circle M = the radius length of the circle N = MN , then the two circles are .... *(Alexandria 05)*

(a) one is inside the other. (b) touching externally.
(c) distant. (d) intersecting.

12 If the two circles M and N are touching internally , the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. *(Giza 17)*

(a) 12 (b) 11 (c) 6 (d) 5

13 M and N are two touching circles where MN = 6 cm. , the radius length of the greater circle is 10 cm. , then the radius length of the smaller circle = ... cm. *(El-Sharkia 05)*

(a) 16 (b) 12 (c) 8 (d) 4

14	M , N and L are three circles touching externally two-by-two, their radii lengths are 5 cm., 6 cm. and 4 cm., then the perimeter of the triangle MNL = ... cm. *(El-Monofia 11)* (a) 15 (b) 30 (c) 4 (d) 60	
15	If the two circles M and N are touching externally , the radius length of the circle M is 4 cm. , if MN = 7 cm. , then the circumference of the circle N is .... cm. *(El-Monofia 16)* (a) 4 π (b) 6 π (c) 7 π (d) π	
16	A circle M of radius length 4 cm. touches a circle N internally , MN = 7 cm. , then the circumference of the circle M : the circumference of the circle N = ... *(El-Dakahlia 09)* (a) 4 : 7 (b) 3 : 4 (c) 4 : 3 (d) 4 : 11	

Solutions

A — Essay Problems

1

$\because MN = MA + NA$

$, \because NA = NB = 7$ cm. (lengths of two radii)

$\therefore 12 = MA + 7 \quad \therefore MA = 5$ cm. (The req.)

2

$\because$ The two circles are touching internally at A

$\therefore MN = 10 - 6 = 4$ cm. $, \overleftrightarrow{MN} \perp \overleftrightarrow{AB}$

$\therefore$ The area of Δ BMN $= \frac{1}{2} \times MN \times AB$

$\therefore 24 = \frac{1}{2} \times 4 \times AB \quad \therefore AB = 12$ cm. (The req.)

3

$\because \overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord

$\therefore \overline{AB} \perp \overline{MN} \qquad \therefore m(\angle AEN) = 90°$

$\because$ The sum of the measures of the interior angles of the quadrilateral CDNE $= 360°$

$\therefore m(\angle CDN) = 360° - (55° + 125° + 90°) = 90°$

$\therefore \overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

4

$\because \overleftrightarrow{NM}$ is the line of centres , $\overline{AB}$ is the common chord

$\therefore \overline{MN} \perp \overline{AB} \qquad \therefore m(\angle AEM) = 90°$

$, \because$ C is the midpoint of $\overline{XY}$

$\therefore \overline{MC} \perp \overline{XY} \qquad \therefore (\angle MCX) = 90°$

In the quadrilateral DCME :

$m(\angle CME) = 360° - (90° + 90° + 40°) = 140°$ (First req.)

$, \because \overleftrightarrow{FZ}$ is a tangent to the circle N at F

$\therefore \overline{NF} \perp \overleftrightarrow{FZ} \qquad \therefore m(\angle NFZ) = 90°$

$\therefore m(\angle MEA) = m(\angle NFZ)$ and they are corresponding angles

$\therefore \overleftrightarrow{FZ} \parallel \overline{AB}$ (Second req.)

5

$\because \overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord of the two circles

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$, C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{1}{2} \times 12 = 6$ cm.

$\therefore MC = \sqrt{(AM)^2 - (AC)^2} = \sqrt{100 - 36} = 8$ cm.

In Δ AMN :

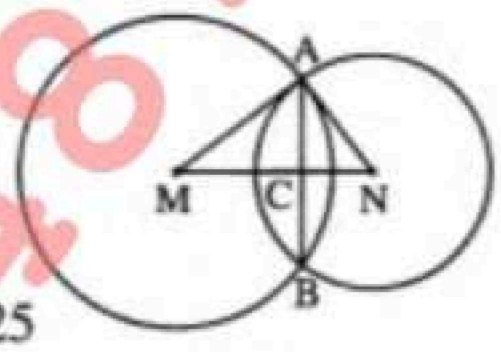
$\because AM = AN = r \qquad , \overline{AC} \perp \overline{MN}$

$\therefore$ C is the midpoint of $\overline{MN}$

$\therefore MN = 2\,MC = 2 \times 8 = 16$ cm. (The req.)

6

$\because \overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord of the two circles



$\therefore \overleftrightarrow{MN} \perp \overline{AB}$, AC = CB

In Δ AMN : $(AN)^2 = 81$

$, (AM)^2 = 144 , (MN)^2 = 225$

$\therefore (MN)^2 = (AM)^2 + (AN)^2$

$\therefore \Delta$ AMN is right-angled at A , $\because \overline{AC} \perp \overline{MN}$

$\therefore AC = \frac{AM \times AN}{MN} = \frac{12 \times 9}{15} = 7.2$ cm.

$\therefore AB = 2\,AC = 14.4$ cm. (The req.)

7

$\because \overleftrightarrow{MN}$ is the line of centres

$, \overline{AB}$ is the common chord

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore CA = CB$

$\therefore$ In Δ ABC : $m(\angle CAB) = m(\angle CBA)$ (1)

$, \because DA = DB$

$\therefore$ In Δ ABD : $m(\angle DAB) = m(\angle DBA)$ (2)

By adding (1) , (2) :

$\therefore m(\angle CAD) = m(\angle CBD)$ (Q.E.D.)

8

$\because MA = \sqrt{(3+1)^2 + (5+3)^2} = \sqrt{16 + 64} = 4\sqrt{5}$ length unit

$\therefore A \in$ the circle M

$, \because NA = \sqrt{(-3+1)^2 + (-7+3)^2} = \sqrt{4 + 16} = 2\sqrt{5}$ length units

$\therefore A \in$ the circle N

$\because MN = \sqrt{(3+3)^2 + (5+7)^2} = \sqrt{36 + 144} = \sqrt{180} = 6\sqrt{5}$ length units

$\therefore MN = MA + NA$

$\therefore$ The two circles are touching externally. (Q.E.D.)

B	Choose
1	D
2	C
3	B
4	D
5	A
6	B
7	C
8	B
9	D
10	C
11	D
12	B
13	D
14	B
15	B
16	D

Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [4] : Identifying The Circle

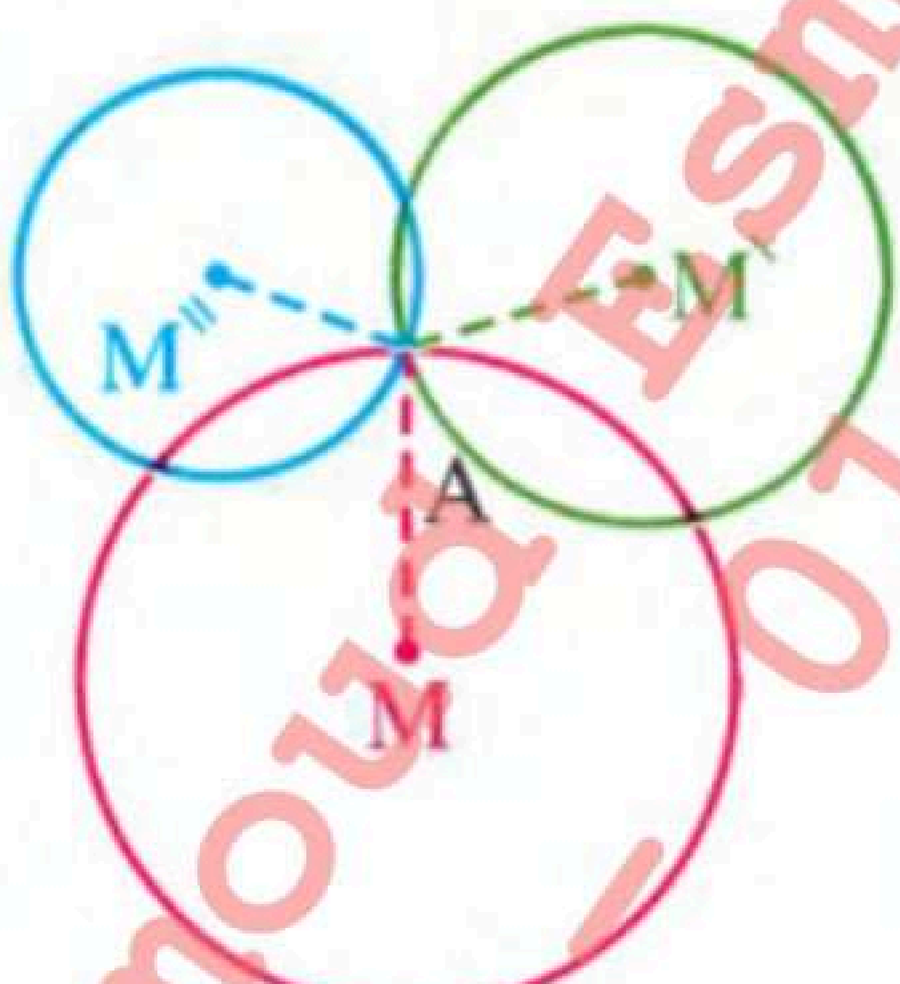
The circle is identified if we know :

1. its centre.
2. its radius length.

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

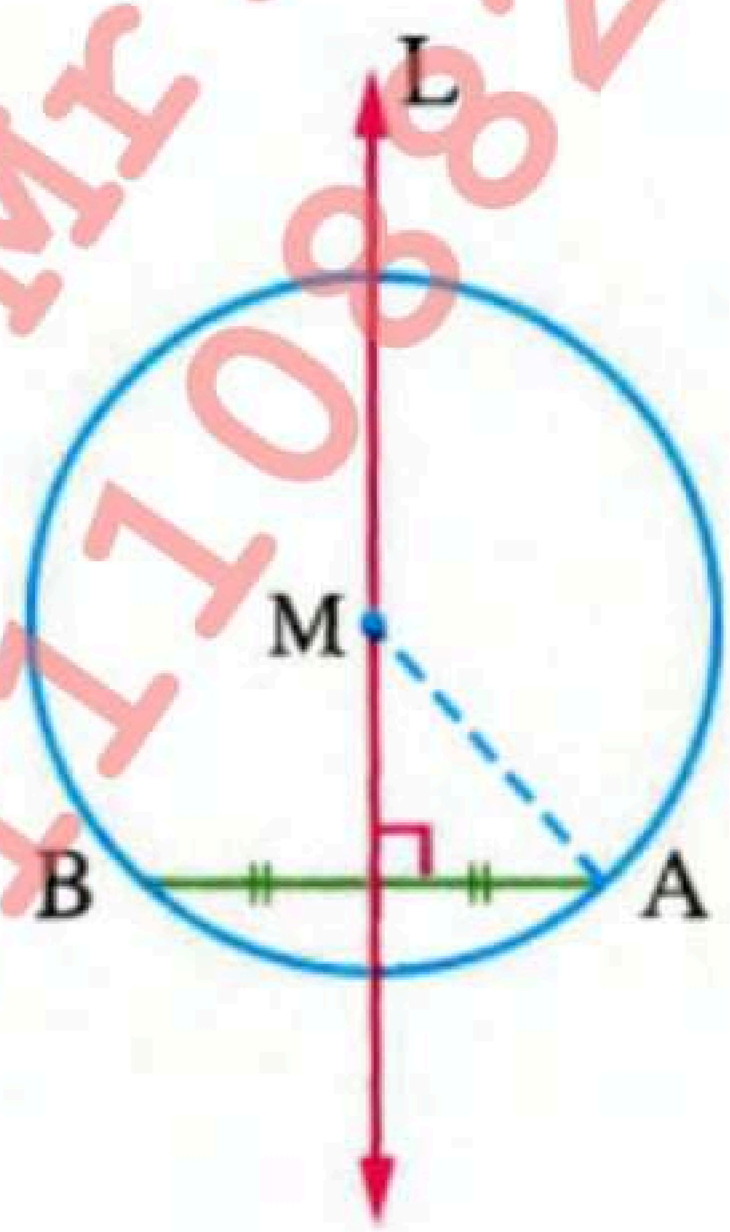
First Drawing a circle passing through a given point

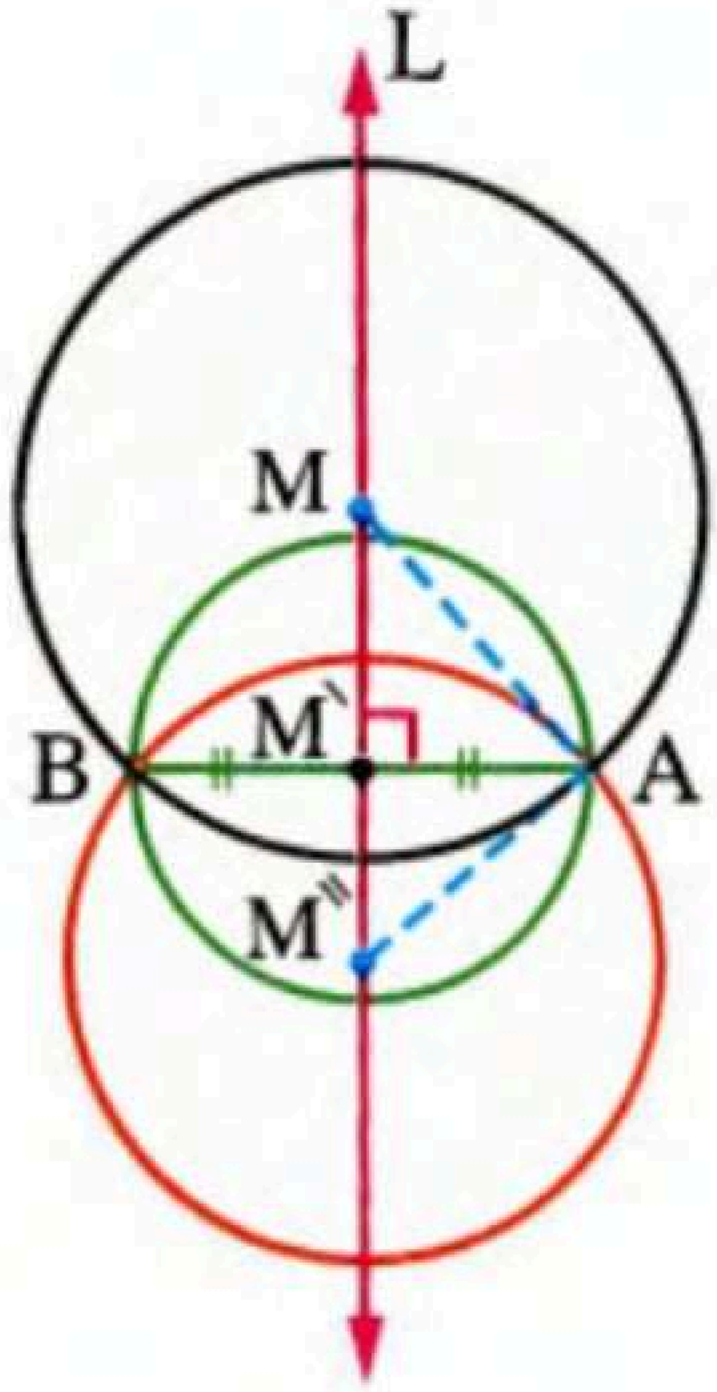
We can draw an infinite number of circles passing through a given point.



Second Drawing a circle passing through two given points

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$





Remarks

- **If $\overline{AB}$ is a line segment and the required is drawing a circle passing through the two points A and B , then :**

1. If $\boxed{r > \frac{1}{2} AB}$, then we can draw two circles (as shown in the previous example).
2. If $\boxed{r = \frac{1}{2} AB}$, then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence $\overline{AB}$ is a diameter of it and its centre is the midpoint of $\overline{AB}$
3. If $\boxed{r < \frac{1}{2} AB}$, then it is impossible to draw any circle.

- **Any two circles do not intersect at more than two points.**

Third Drawing a circle passing through three given points

If A , B and C are three points in the plane and the required is drawing a circle passing through the three points A , B and C :

Then we must distinguish between two cases :

1 If the points A , B and C are collinear as in figure (1)

, then the two straight lines L_1 and L_2 are parallel not intersecting.

In this case , it is impossible to draw a circle passing through the three points A , B and C

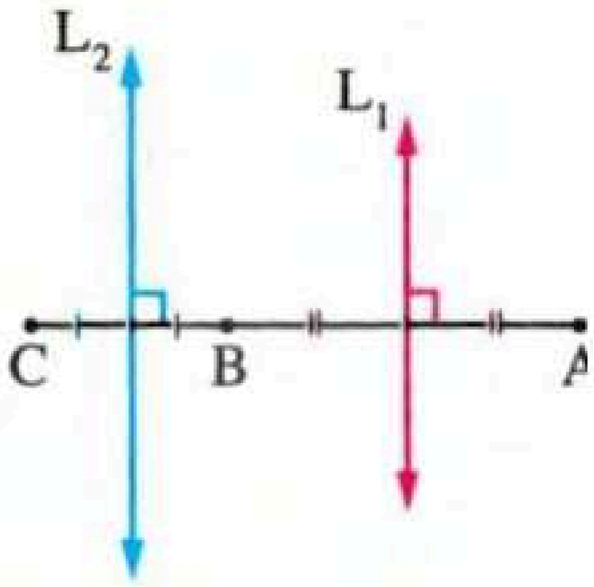


Fig (1)

i.e.

It is impossible to draw a circle passing through three collinear points.

2 If the points A , B and C are not collinear as in figure (2) ,

then L_1 and L_2 intersect at one point as M , then M is the centre of the required circle which passes through the three points A , B and C , then the radius length of this circle = MA = MB = MC

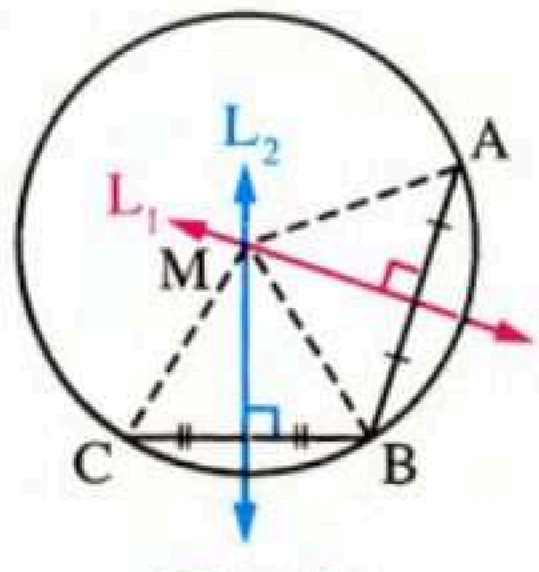


Fig (2)

i.e.

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

Notice that :

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

Corollary 1

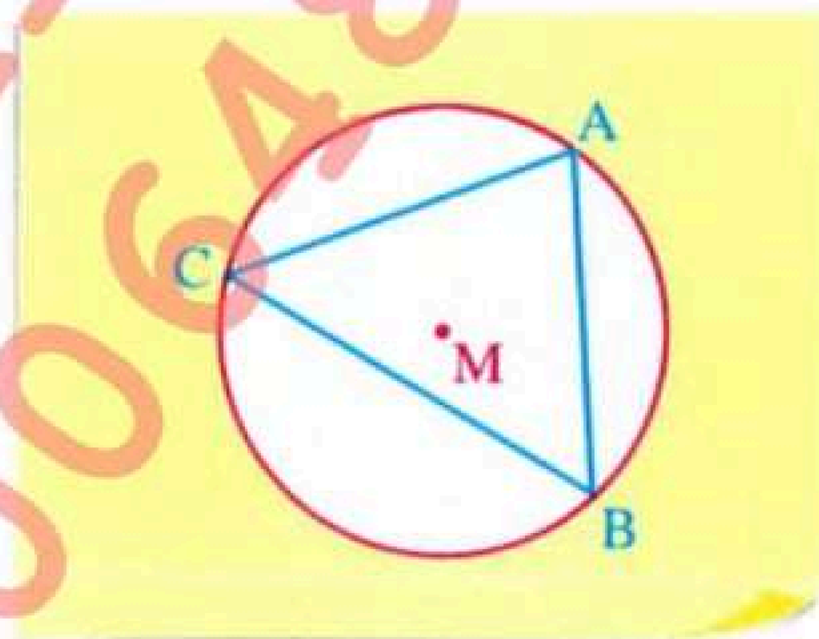
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

M is the circumcircle of Δ ABC

or Δ ABC is the inscribed triangle of the circle M



Corollary 2

The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

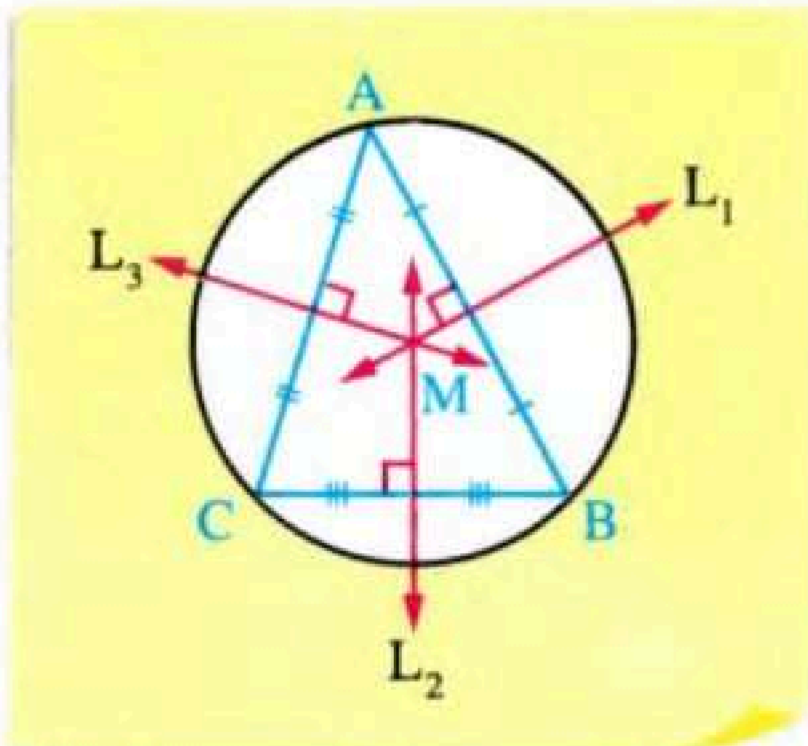
In the opposite figure :

If the straight lines L_1 , L_2 and L_3

are the axes of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively

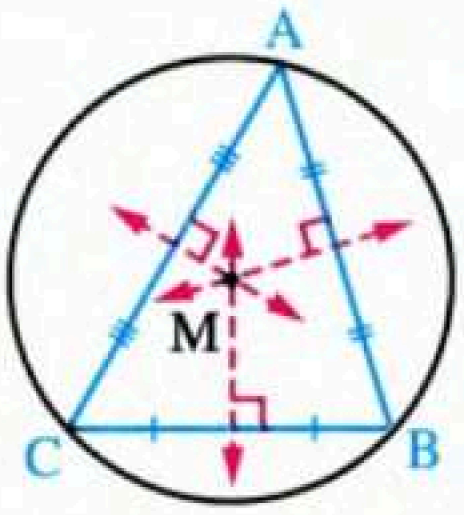
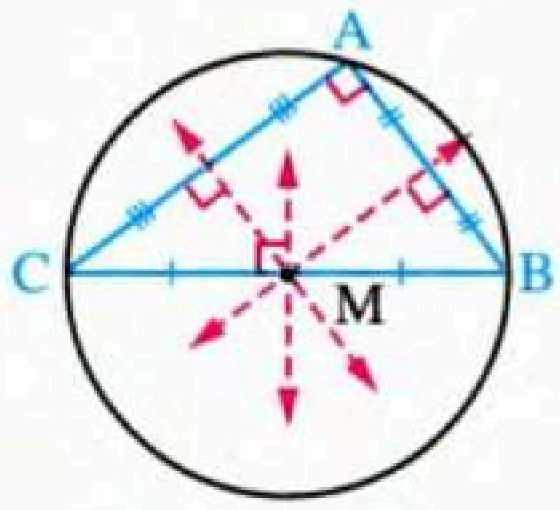
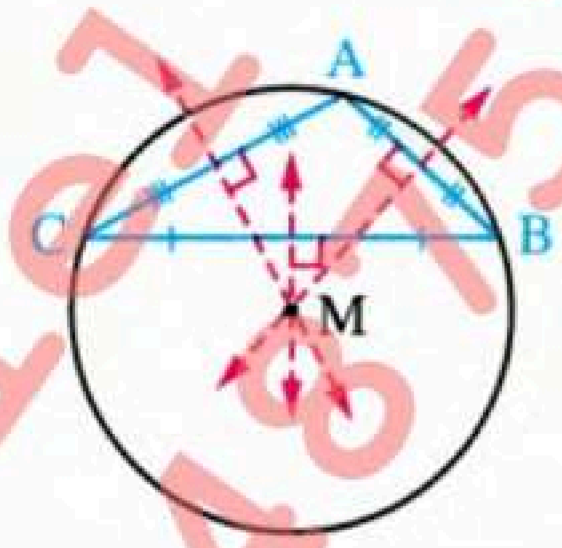
and $L_1 \cap L_2 \cap L_3 = \{M\}$,

then the point M is the centre of the circumcircle of Δ ABC



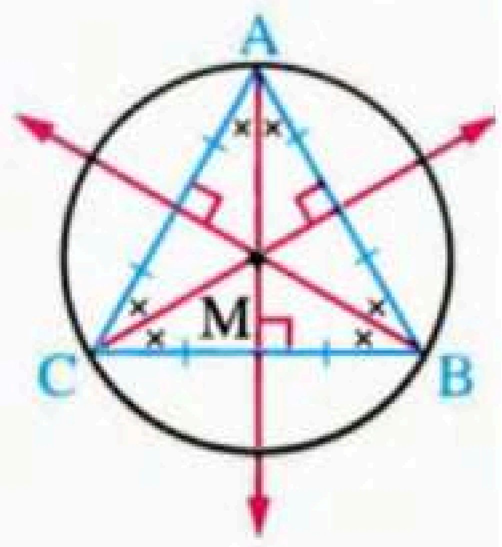
Remark

The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :

The acute-angled triangle	The right-angled triangle	The obtuse-angled triangle
		
M is inside the triangle	M is the midpoint of the hypotenuse	M is outside the triangle

- **A special case :**

The centre of the circumcircle of the equilateral triangle is :



- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its interior angles.

Remark

We can draw a circle passing through the vertices of (the rectangle , the square or the isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram , the rhombus or the trapezium which is not isosceles).

Exercises

[A] Essay problems : -

1. **If $A \in L$, draw the circle M passing through A and its radius length = 3 cm. if :**

1. $M \in$ the straight line L , how many circles can be drawn ?
2. $M \notin$ the straight line L , how many circles can be drawn ? *(Assiut 11)*

2. A and B are two points where AB = 6 cm. Draw a circle of radius length 5 cm. and passes through the two points A and B

Find :

1. The number of circles can be drawn.
2. The distance of the centre of the circle from $\overline{AB}$ by proof. *(Damietta 17)* « 4 cm. »

3. $\overline{AB}$ is a line segment of length 6 cm. Draw the circle that passes through the two points A and B and its radius length is the smallest length. *(Luxor 05)*

4. Using the geometric tools and draw $\overline{AB}$ with length 6 cm. , then draw $\overrightarrow{AC}$ where m (∠ CAB) = 60° , draw the circle that passes through the points A , B and its centre lies on $\overrightarrow{AC}$ and calculate the length of its radius (Don't remove the arcs). *(El-Dakahlia 17)* « 6 cm. »

5. Draw a circle with radius length of 3 cm. and touches to the straight line L What is the number of possible solutions ? *(Giza 06)*

6. Draw the right-angled triangle ABC at B where AB = 4 cm. and BC = 3 cm., then draw the circumcircle of this triangle. Where does the centre of the circle lie with respect to the sides of this triangle ? *(Damietta 18)*

7. Using geometrical instruments, draw the isosceles triangle ABC in which m (∠ ABC) = 120° , BC = 4 cm. Determine the centre of the circumcircle of it and find its radius length. *(El-Dakahlia 11)* « 4 cm. »

8. Draw Δ ABC in which : AB = 5 cm. , BC = 4 cm. , and CA = 3 cm. What is the type of the triangle with respect to the measures of its angles ? then draw a circle whose centre is the point A and touches $\overleftrightarrow{BC}$, another circle whose centre is B and touches $\overleftrightarrow{AC}$ and a third circle whose centre is C and touches $\overleftrightarrow{AB}$ *(Beni Suef 06)*

9 If A (2 , 0) and B (– 2 , 3) , draw a circle M of radius length 4 length units and passes through the two points A and B

How many solutions are there for this problem ? *(North Sinai 09)*

[B] Choose the correct : -

1	It is possible to draw .... passing through a given point. *(New Valley 05)* (a) one circle (b) two circles (c) three circles (d) an infinite number of circles
2	The number of circles which passes through two given points is .... *(Giza 12)* (a) 1 (b) 2 (c) 3 (d) an infinite number.
3	The number of circles passing through three collinear points is .... *(Souhag 18 , Giza 16 , Ismailia 15)* (a) zero (b) one (c) three (d) an infinite number.
4	The number of circles passing through three non-collinear points is .... *(El-Menia 17)* (a) 1 (b) zero (c) 2 (d) 3
5	We can identify the circle if we are given .... *(El-Sharkia 08)* (a) three collinear points. (b) two points. (c) three non-collinear points. (d) one point.
6	The centres of the circles passing through the two points A and B lie on .... *(El-Dakahlia 17)* (a) the axis of symmetry of $\overline{AB}$ (b) $\overline{AB}$ (c) the perpendicular to $\overline{AB}$ (d) the midpoint of $\overline{AB}$
7	The centre of the circumcircle of a triangle is the point of intersection of .... *(El-Fayoum 19 , Kafr El-Sheikh 17 , Qena 17)* (a) the bisectors of its interior angles. (b) the bisectors of its exterior angles. (c) its altitudes. (d) the symmetry axes of its sides.

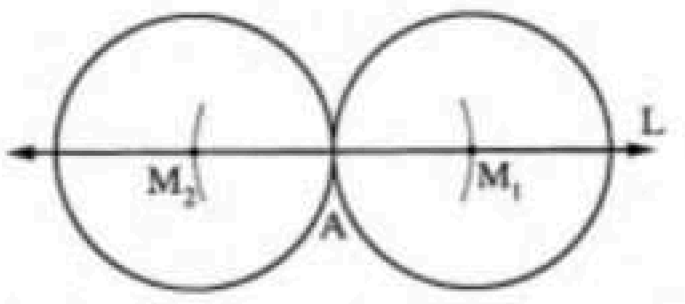
8	If ΔABC is right-angled at B , then the centre of its circumcircle is .... *(Ismailia 03)* (a) the midpoint of $\overline{AB}$ (b) the midpoint of $\overline{AC}$ (c) the midpoint of $\overline{BC}$ (d) outside the triangle.
9	It is (impossible) to draw a circle passing through the vertices of .... *(Beni Suef 17 , El-Dakahlia 13 , El-Sharkia 12)* (a) a rectangle. (b) a triangle. (c) a square. (d) a rhombus.
10	It is possible to draw a circle passing through the vertices of .... *(El-Sharkia 19 , Souhag 18 , Giza 17 , Beni Suef 16)* (a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a parallelogram.
11	If $\overline{AB}$ is a line segment of length 4 cm. , then the radius length of the smallest circle which passes through the two points A and B = ... cm. *(El-Monofia 16)* (a) 2 (b) 3 (c) 4 (d) 5
12	If AB = 6 cm. , then the area of the smallest circle which passes through the two points A and B = ... cm^2. *(El-Sharkia 15)* (a) 3π (b) 6π (c) 8π (d) 9π

Solutions

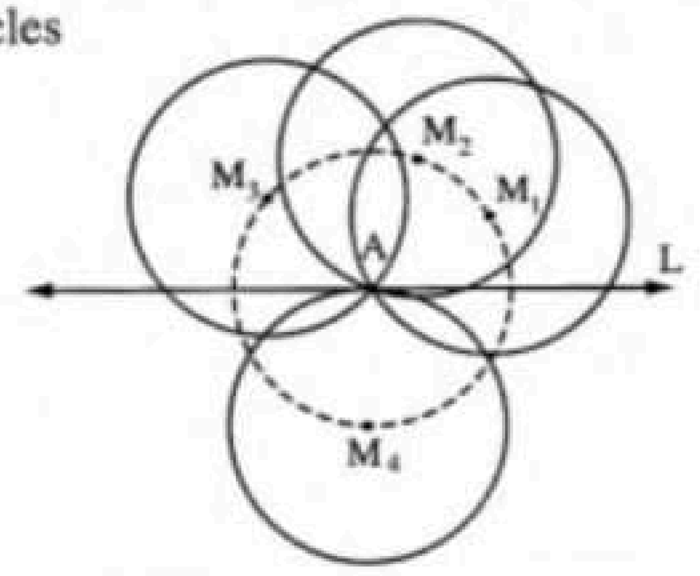
A Essay Problems

1

1 When $M \in L$ we can draw two circles



2 When $M \notin L$ we can draw an infinite number of circles



2

1 We can draw two circles

2 In ΔAM_1D :

$\because \overline{M_1D} \perp \overline{AB}$

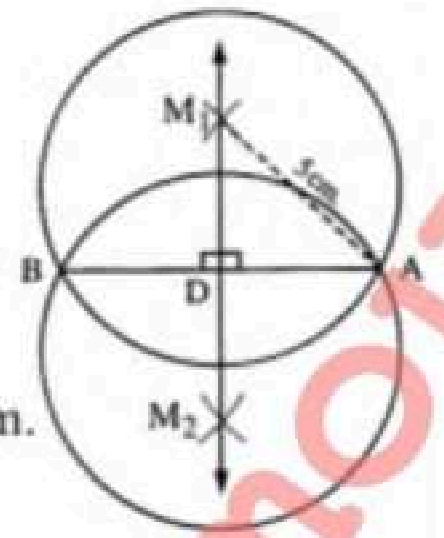
$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$, \because m(\angle ADM_1) = 90°$

$\therefore M_1D = \sqrt{(AM_1)^2 - (AD)^2}$

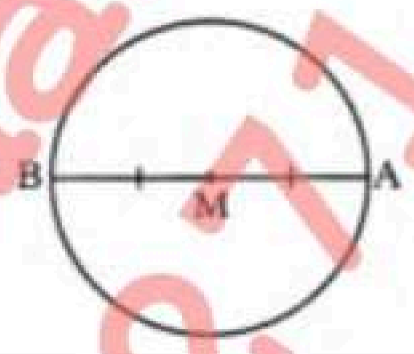
$= \sqrt{25 - 9} = 4$ cm. (The req.)



3

$\because$ The radius length is the smallest

$\therefore r = 3$ cm.



4

In ΔADM : $\because \overline{AD} \perp \overline{MD}$

$\therefore$ D is the midpoint of $\overline{AB}$

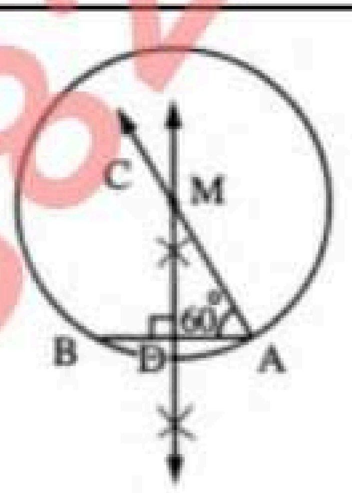
$\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$, \because m(\angle ADM) = 90°$

$, m(\angle A) = 60°$

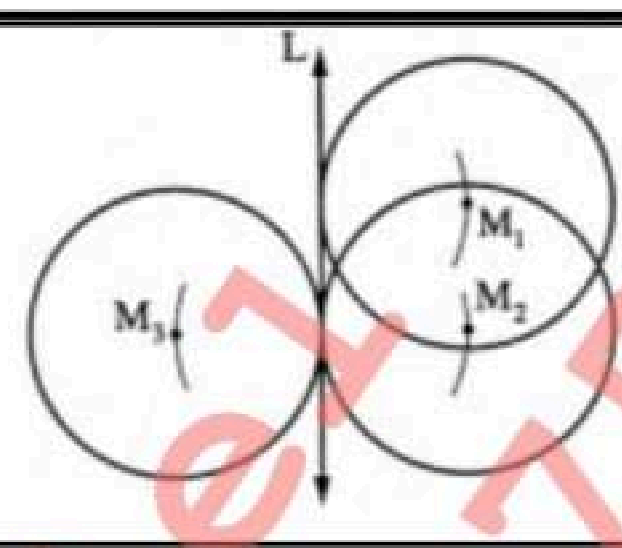
$\therefore m(\angle AMD) = 180° - (90° + 60°) = 30°$

$\therefore AM = 2\,AD = 2 \times 3 = 6$ cm. (The req.)



5

There are an infinite number of circles whose centres lie on a straight line parallel to the straight line L at a distance 3 cm. from it.



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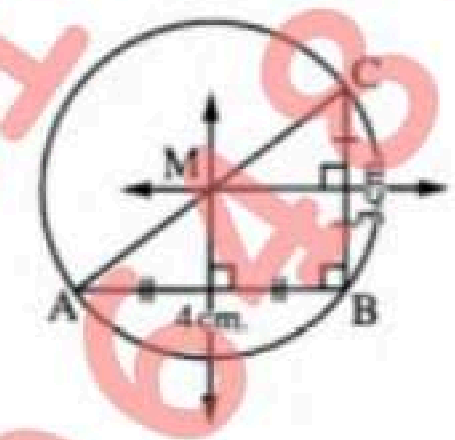


The centre of the circle lies at the midpoint of the hypotenuse $\overline{AC}$

7

In ΔABC :

$\because AB = BC$

$\therefore \Delta ABC$ is an isosceles triangle

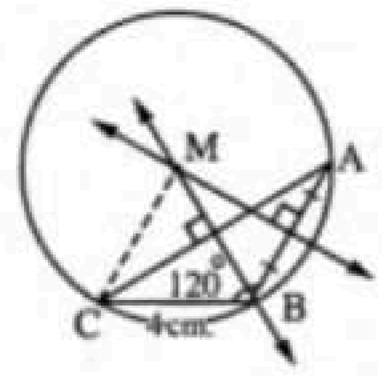
$, \because \overline{BM} \perp \overline{AC}$

$\therefore \overrightarrow{BM}$ bisects $\angle ABC$ $\therefore m(\angle MBC) = 60°$

$, \because MB = MC = r$

$\therefore \Delta MBC$ is an equilateral triangle

$\therefore MB = MC = BC = r = 4$ cm. (The req.)



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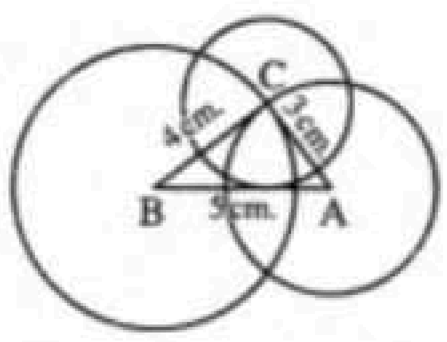


The type of this triangle according to the measures of its angle is right-angled triangle at C

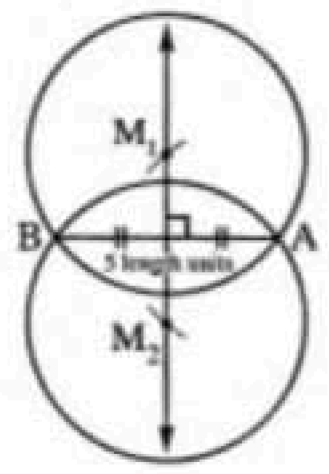
9

$AB = \sqrt{(2+2)^2 + (0-3)^2}$

$= \sqrt{16 + 9} = \sqrt{25}$

$= 5$ length units

$\therefore$ There are two solutions



B Choose

1 D

2	D
3	A
4	A
5	C
6	A
7	D
8	B
9	D
10	B
11	A
12	D

Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [5] : The Relation Between The Chords Of A Circle With Its Center

Fig. (1)

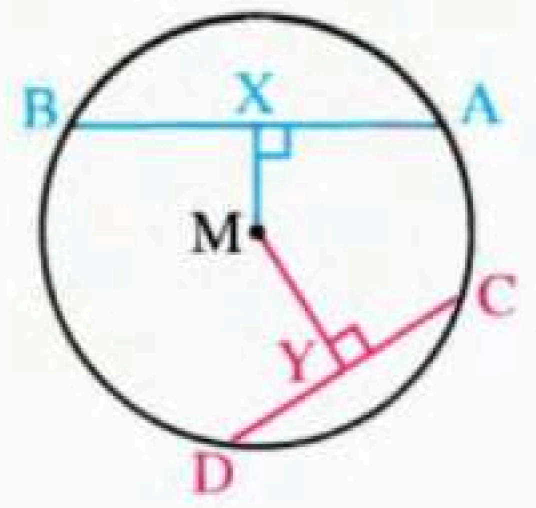


Fig. (2)

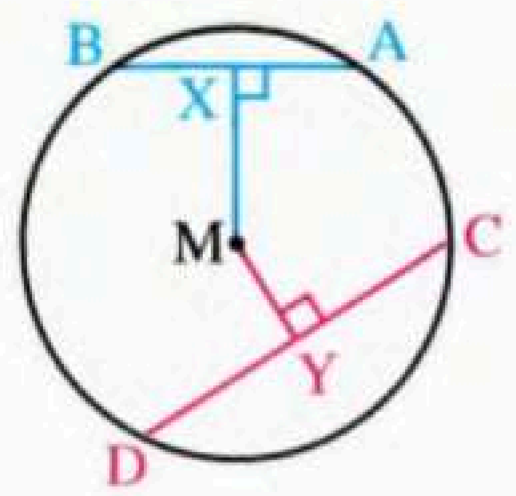
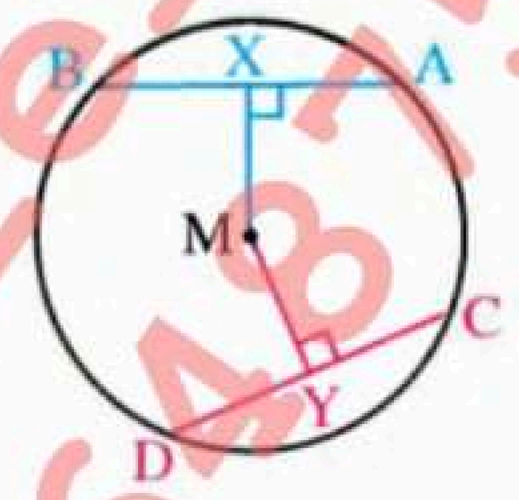


Fig. (3)



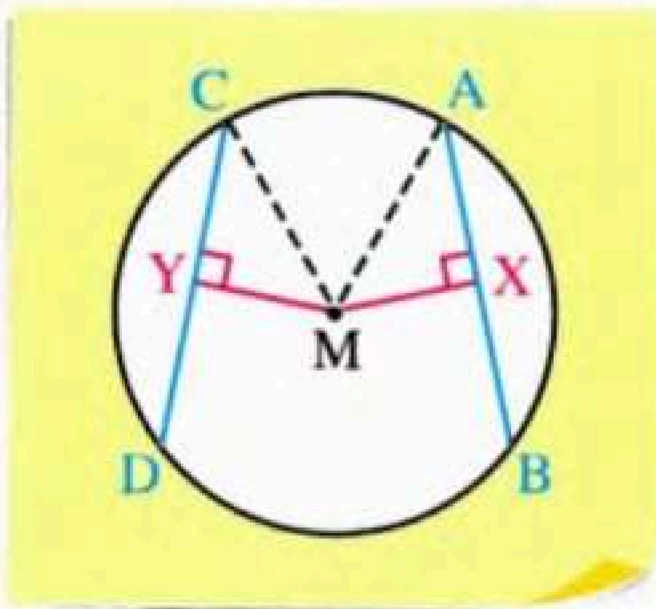
Using the ruler , you can check by yourself the truth of the following information :

Fig. (1)	Fig. (2)	Fig. (3)
AB > CD , MX < MY	AB < CD , MX > MY	AB = CD , MX = MY

The relation between the chords of a circle and its centre :

Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.



Given $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

R.T.P. $MX = MY$

Construction Draw $\overline{MA}$ and $\overline{MC}$

Proof $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$

$\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$

$\because AB = CD$ (given) $\therefore AX = CY$

$\because \Delta\Delta$ AXM and CYM , both have $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90° \end{cases}$

$\therefore \Delta AXM \equiv \Delta CYM$, then we get : $MX = MY$ (Q.E.D.)

Corollary

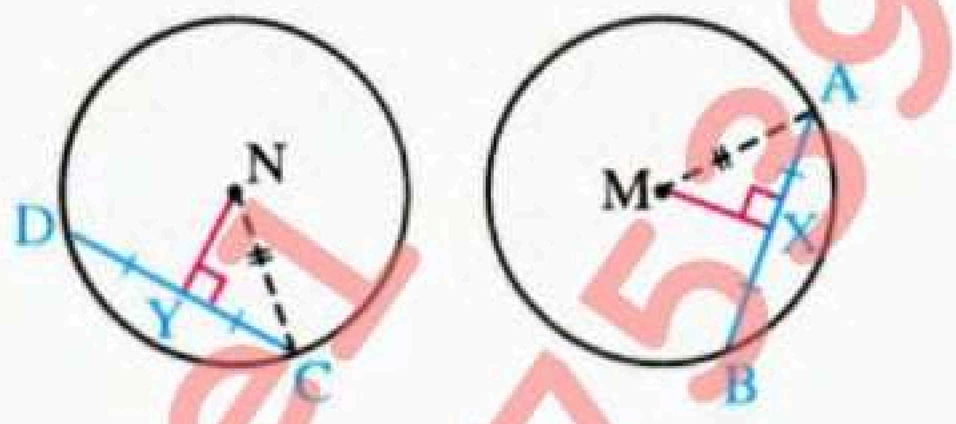
In congruent circles, chords which are equal in length are equidistant from the centres.

In the opposite figure :

If M and N are two congruent circles ,

AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$,

then MX = NY



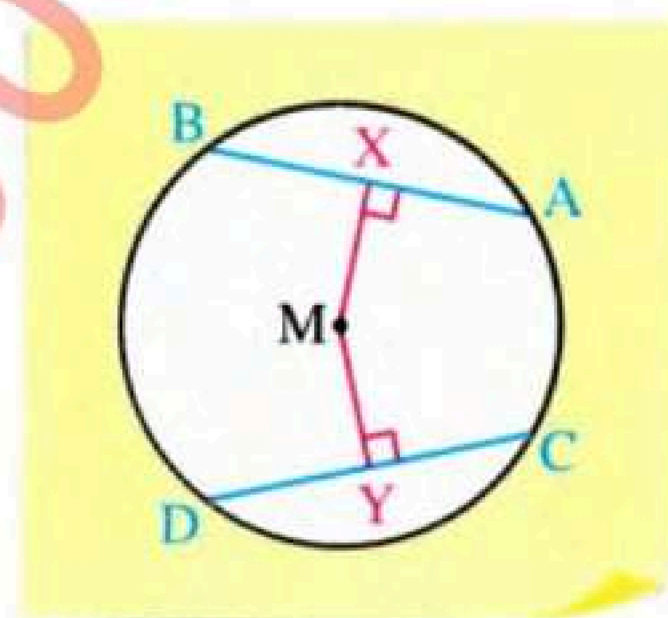
Converse of the theorem

In the same circle (or in congruent circles) ,
chords which are equidistant from the centre (s) are equal in length.

i.e. **In the opposite figure :**

If $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ and MX = MY , then AB = CD

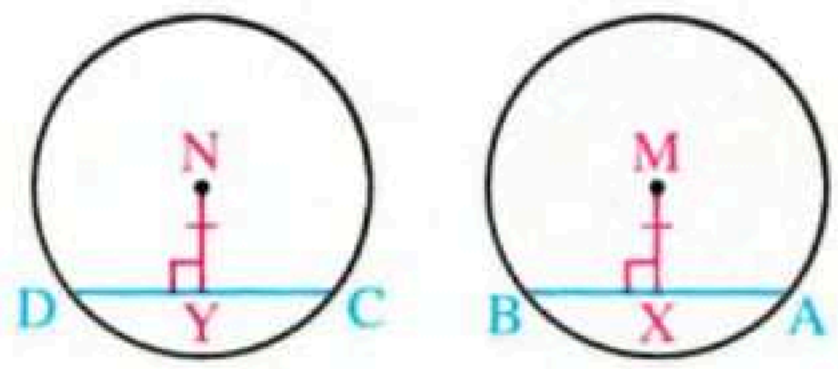


Also in the opposite figure :

If M and N are two congruent circles , $\overline{AB}$ is a chord of circle M and $\overline{CD}$ is a chord of circle N

, $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$ and

MX = NY , then AB = CD



Exercises

[A] Essay problems : -

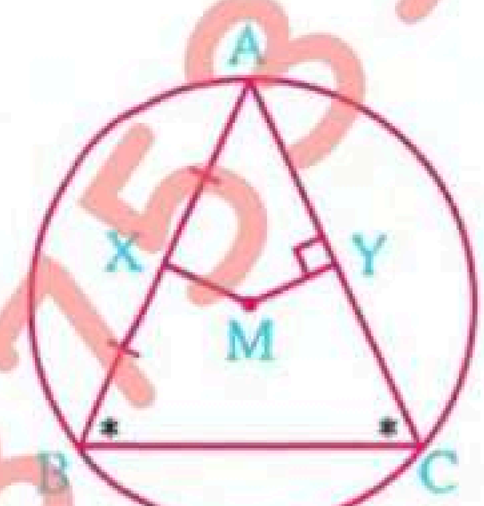
1

In the opposite figure :

The triangle ABC is an inscribed triangle inside a circle M ,

m (∠ B) = m (∠ C) ,

X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$



Prove that : MX = MY *(Giza 19 , El-Beheira 19 , Matrouh 17 , Fayoum 15)*

2

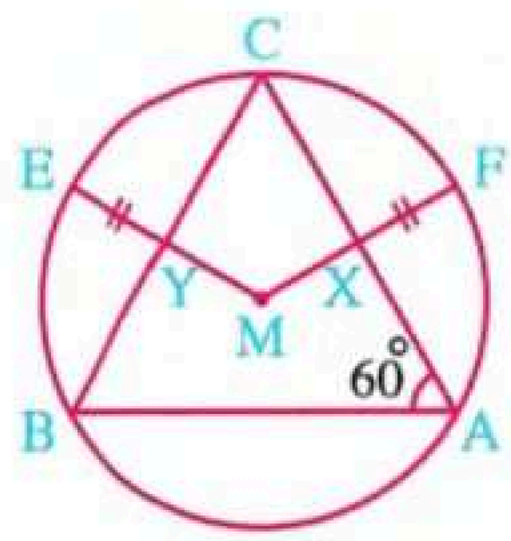
In the opposite figure :

M is a circle , m (∠ A) = 60°

, X is the midpoint of $\overline{AC}$

, Y is the midpoint of $\overline{BC}$

, FX = EY



Prove that : Δ ABC is an equilateral triangle *(El-Sharkia 18)*

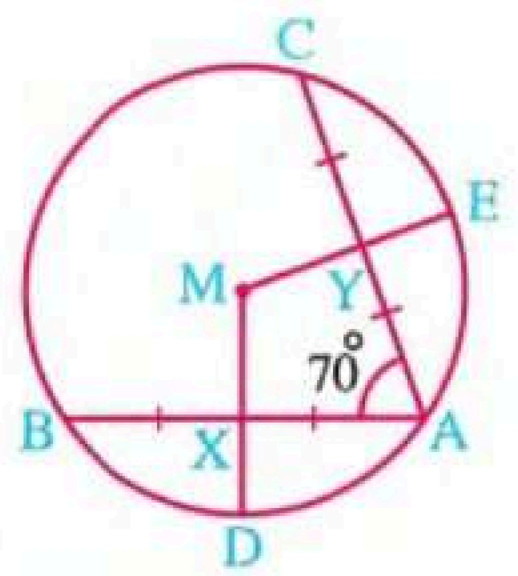
3

In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and m (∠ CAB) = 70°



1 **Calculate :** m (∠ DME) « 110°»

2 **Prove that :** XD = YE *(New Valley 19 , Port said 18 , Matrouh 18 , Cairo 17)*

4

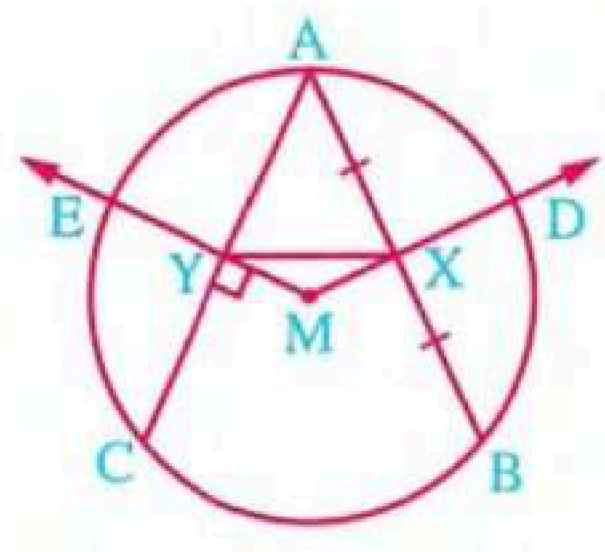
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$,

$\overrightarrow{MX}$ intersects the circle at D , $\overrightarrow{MY} \perp \overline{AC}$

intersects it at Y and intersects the circle at E



Prove that : **1** XD = YE **2** m (∠YXB) = m (∠ XYC) *(Assiut 18 , El-Gharbia 13)*

5

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , m (∠ MXY) = 30°

Prove that : **1** Δ MXY is an isosceles triangle.

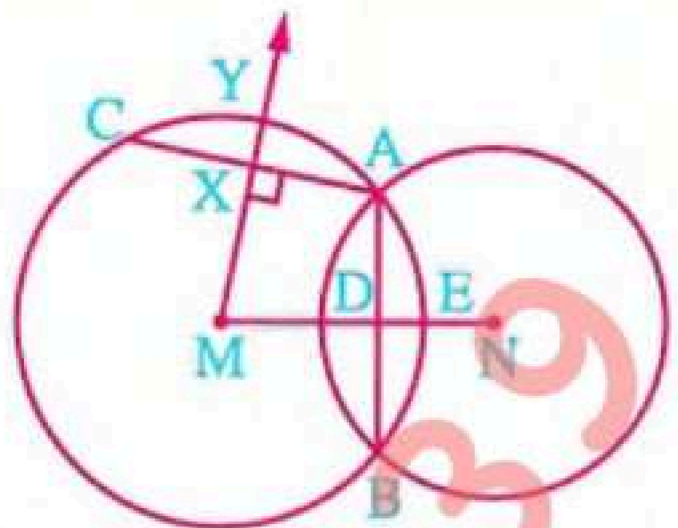
2 Δ AXY is an equilateral triangle. *(New Valley 16)*

6

In the opposite figure :

M and N are two circles intersecting at A and B , $\overrightarrow{MX} \perp \overline{AC}$ and intersects $\overline{AC}$ at X and intersects the circle M at Y , $\overline{MN}$ to intersects $\overline{AB}$ at D and intersects the circle M at E, if AC = AB

Prove that : XY = DE



(El-Kalyoubia 18)

7

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M , $\overline{MX} \perp \overline{AB}$, Y is the midpoint of $\overline{AC}$, m (∠ ABC) = 75° , MX = MY

1 **Find :** m (∠ BAC) « 30° »

2 **Prove that :** The perimeter of Δ AXY = $\frac{1}{2}$ the perimeter of Δ ABC

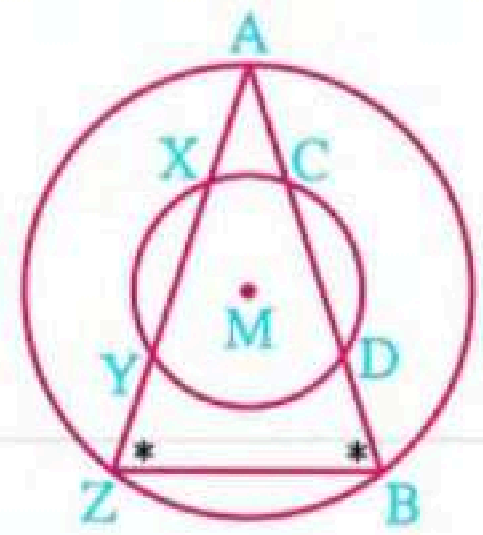
(Kafr-El-Sheikh 18 , Alexandria 16)

8

In the opposite figure :

Two concentric circles at M , $\overline{AB}$ is a chord in the greater circle and cuts the smaller circle at C and D , $\overline{AZ}$ is a chord in the greater circle and cuts the smaller circle at X and Y If m (∠ ABZ) = m (∠ AZB)

Prove that : CD = XY



(El-Kalyoubia 17 , Souhag 13)

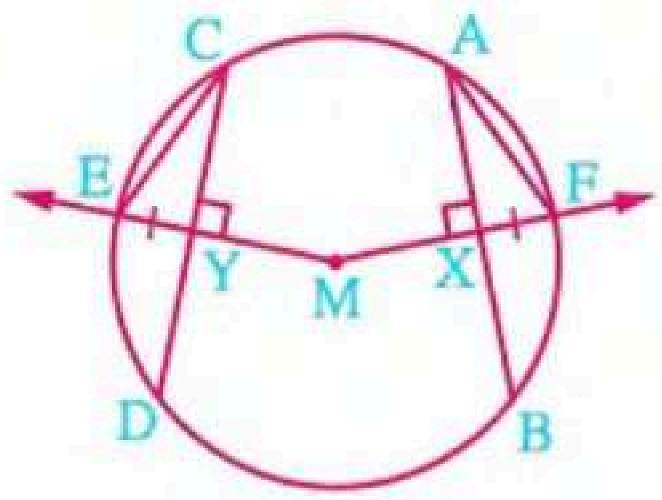
9

In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F , $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that :

1 AB = CD 2 AF = CE



(El-Gharbia 16 , Kafr El-Sheikh 11)

10

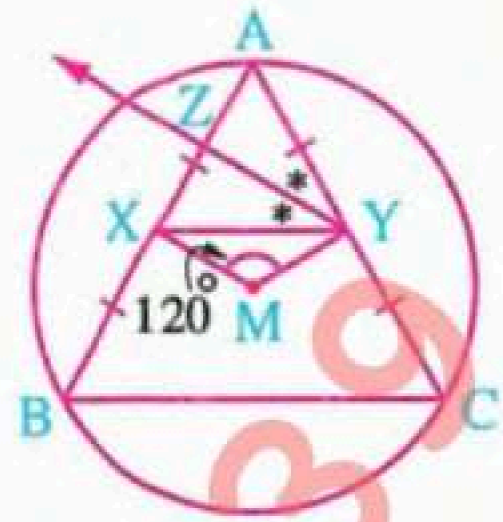
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M , equal in length

X and Y are their midpoints respectively.

If m ($\angle$ XMY) = 120° , $\overrightarrow{YZ}$ bisects $\angle$ AYX

Prove that : $\overleftrightarrow{YZ} \parallel \overleftrightarrow{MX}$ *(Cairo 08)*



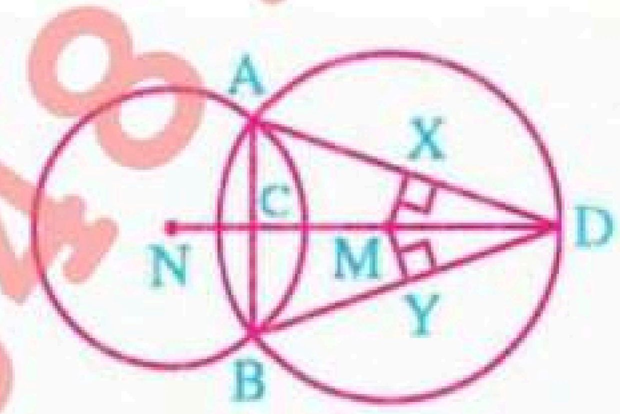
11

In the opposite figure :

The circle M $\cap$ the circle N = $\{A , B\}$, $\overleftrightarrow{AB} \cap \overleftrightarrow{MN} = \{C\}$,

D $\in \overleftrightarrow{MN}$, $\overline{MX} \perp \overline{AD}$ and $\overline{MY} \perp \overline{BD}$

Prove that : MX = MY *(El-Kalyoubia 19 , El-Sharkia 11)*



12

In the opposite figure :

The concentric circles of radii 4 cm. , 2 cm.

Δ ABC is drawn such that its vertices lie

on the greater circle and its sides touch

the smaller circle at X , Y , Z

Prove that :

Δ ABC is an equilateral triangle and find its area. *(El-Fayoum 19)* « $12\sqrt{3}$ cm^2. »

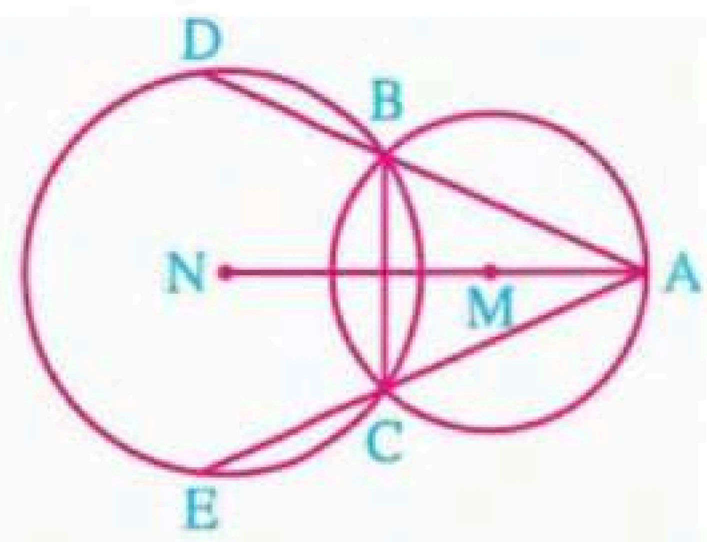
13

In the opposite figure :

M , N are two intersecting circles at B , C

, A $\in \overleftrightarrow{MN}$

Prove that : BD = CE *(El-Dakahlia 17)*



14

In the opposite figure :

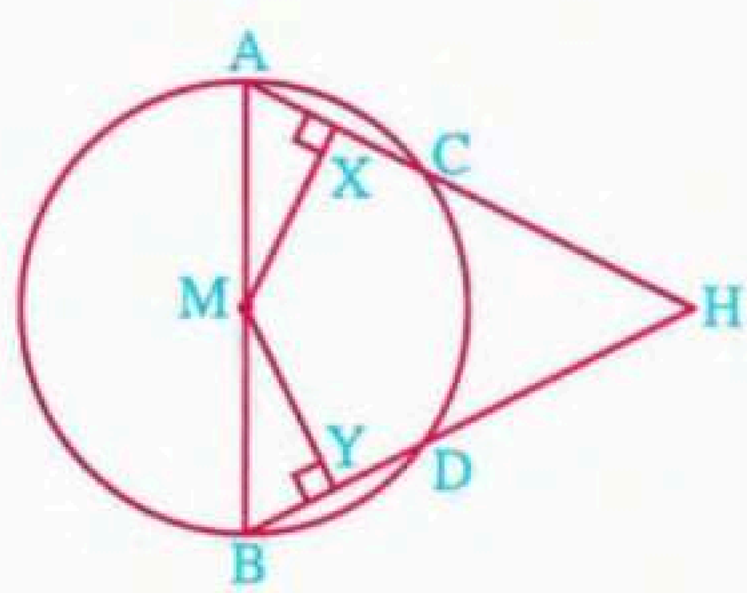
$\overline{AB}$ is a diameter of the circle M , $\overline{AC}$ and $\overline{BD}$ are two chords in it ,

MX = MY , $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{DB}$

Prove that :

1 Δ HAB is isosceles triangle.

2 HC = HD *(Beni Suef 12)*



15

In the opposite figure :

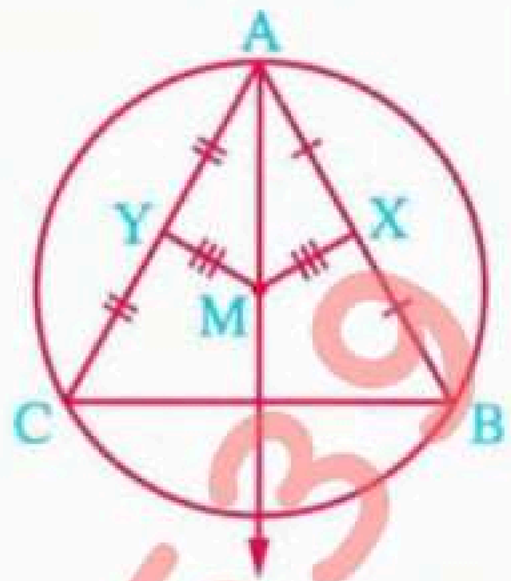
Δ ABC is inscribed in the circle M ,

m ($\angle$ BAC) = 60° , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and MX = MY

Prove that :

1 ABC is an equilateral triangle. 2 $\overrightarrow{AM} \perp \overline{BC}$



(Giza 05)

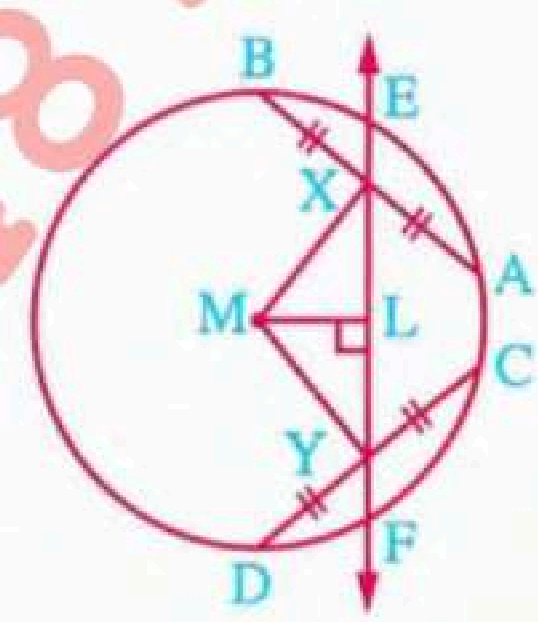
16

In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

equal in length , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. $\overleftrightarrow{XY}$ is drawn to cut the circle at E and F , $\overline{ML}$ is drawn $\perp$ $\overleftrightarrow{XY}$

Prove that : XE = YF



(Cairo 03)

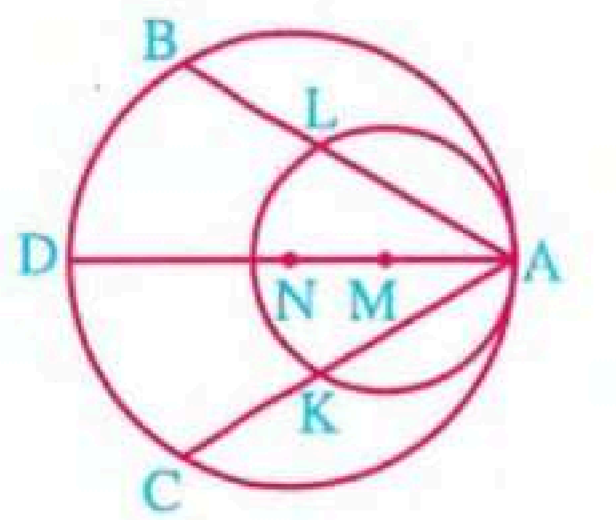
17

In the opposite figure :

M and N are two circles touching internally at A ,

$\overline{AB}$ and $\overline{AC}$ are two chords drawn in the greater circle N such that they are equal in length to cut the smaller circle M at L and K respectively.

Prove that : AL = AK



(Dakahlia 09)

18

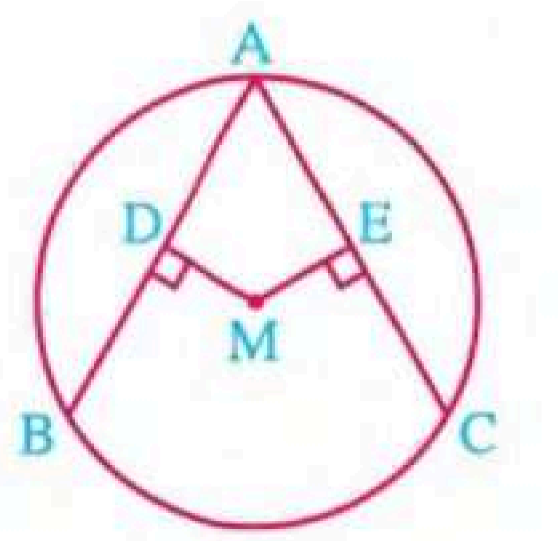
In the opposite figure :

M is a circle , $\overline{MD} \perp \overline{AB}$

, $\overline{ME} \perp \overline{AC}$

, A (2 , 2) , D (1 , 0) and E (3 , 4)

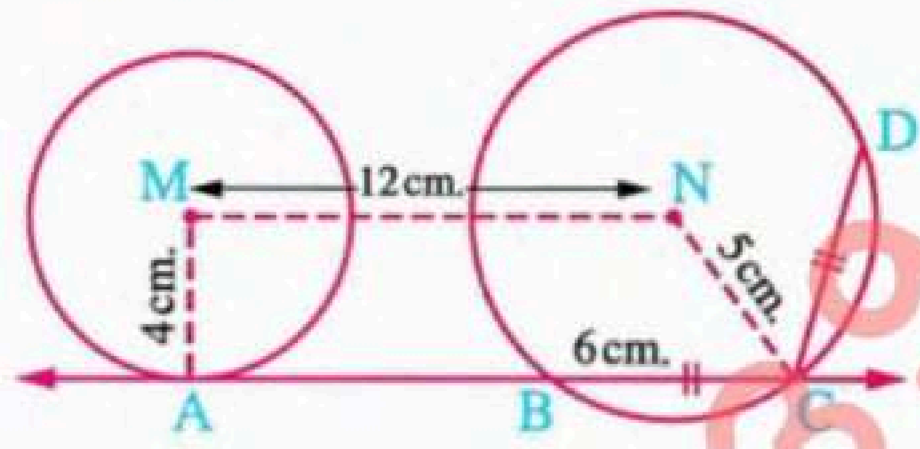
Prove that : ME = MD



(Kafr El-Sheikh 13)

19

In the opposite figure :



M and N are two circles of radii lengths 4 cm. and 5 cm. , $\overline{AC}$ touches the circle M at A and cuts the circle N at B and C , where BC = 6 cm. and MN = 12 cm.

1 Prove that the quadrilateral MACN is a trapezium then calculate its area. « 54 cm^2 »

2 If CD = CB , find the distance between N and $\overline{CD}$ (Sharkia 06) « 4 cm. »

Solutions

A Essay Problems

1

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ $\overline{MY} \perp \overline{AC}$, AB = AC $\therefore$ MX = MY (Q.E.D.)

2

$\because$ MF = ME (lengths of two radii)

, FX = EY

By subtracting : $\therefore$ MX = MY

, $\because$ X is the midpoint of $\overline{AC}$ $\therefore$ $\overline{MX} \perp \overline{AC}$

, $\because$ Y is the midpoint of $\overline{BC}$ $\therefore$ $\overline{MY} \perp \overline{BC}$

$\therefore$ AC = BC , $\because$ m ($\angle$ A) = 60°

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

3

$\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore$ $\overline{MY} \perp \overline{AC}$

$\because$ The sum of measures of the interior angles of the quadrilateral AXMY = 360°

$\therefore$ m ($\angle$ XMY) = 360° – (70° + 90° + 90°) = 110° (First req.)

$\because$ AB = AC $\therefore$ MX = MY

$\because$ MD = ME (lengths of two radii)

by subtracting $\therefore$ XD = YE (Second req.)

4

$\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$

$\because$ AB = AC $\therefore$ MX = MY ,

$\because$ MD = ME (lengths of two radii) by subtracting

$\therefore$ XD = YE (Q.E.D. 1)

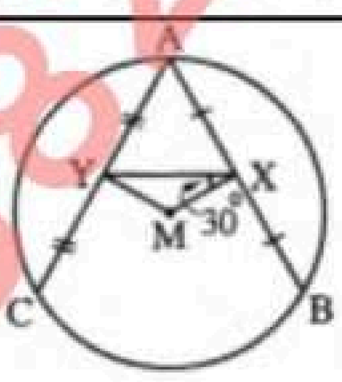
In Δ XMY : $\because$ MX = MY

$\therefore$ m ($\angle$ MXY) = m ($\angle$ MYX)

$\because$ m ($\angle$ MXB) = m ($\angle$ MYC) = 90°

by adding $\therefore$ m ($\angle$ YXB) = m ($\angle$ XYC) (Q.E.D. 2)

5



$\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MY} \perp \overline{AC}$ $\because$ AB = AC

$\therefore$ MX = MY

$\therefore$ Δ MXY is an isosceles triangle (Q.E.D. 1)

$\because$ m ($\angle$ AXM) = 90° , m ($\angle$ MXY) = 30°

$\therefore$ m ($\angle$ AXY) = 90° – 30° = 60°

$\because$ X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$, AB = AC

$\therefore$ AX = AY

$\therefore$ Δ AXY is an equilateral triangle (Q.E.D. 2)

6

$\because$ $\overline{AB}$ is the common chord of the two circles M , N

, $\overleftrightarrow{MN}$ is the line of centres

$\therefore$ $\overleftrightarrow{MN} \perp \overline{AB}$ $\therefore$ $\overline{MD} \perp \overline{AB}$

, $\because$ $\overleftrightarrow{MX} \perp \overline{AC}$, AC = AB

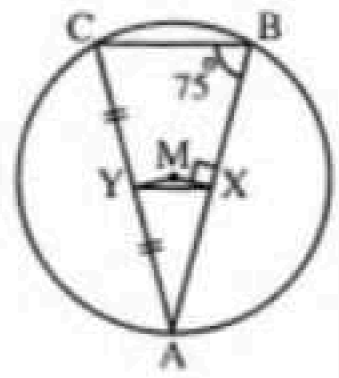
$\therefore$ MX = MD (1)

, $\because$ MY = ME (lengths of two radii) (2)

Subtracting (1) from (2) :

$\therefore$ XY = DE (Q.E.D.)

7



$\because$ Y is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MY} \perp \overline{AC}$

, $\because$ $\overline{MX} \perp \overline{AB}$, MX = MY

$\therefore$ AB = AC

$\therefore$ m ($\angle$ C) = 75°

$\therefore$ m ($\angle$ A) = 180° – (75° + 75°) = 30° (First req.)

$\because$ $\overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

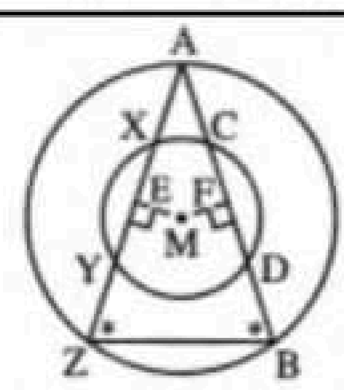
$\therefore$ In Δ ABC :

XY = $\frac{1}{2}$ BC , AX = $\frac{1}{2}$ AB , AY = $\frac{1}{2}$ AC

$\therefore$ The perimeter of Δ AXY

= $\frac{1}{2}$ The perimeter of Δ ABC (Second req.)

8



Constr. :

Draw : $\overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$

Proof : In the great circle :

$\because$ m ($\angle$ ABZ) = m ($\angle$ AZB)

$\therefore$ AB = AZ

$\because$ $\overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$ $\therefore$ MF = ME

In the small circle :

$\because$ $\overline{MF} \perp \overline{CD}$, $\overline{ME} \perp \overline{XY}$, MF = ME

$\therefore$ CD = XY (Q.E.D.)

9

$\because$ MF = ME (lengths of two radii)

, XF = YE $\therefore$ MX = MY

$\because$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ $\therefore$ AB = CD (Q.E.D. 1)

$\because$ $\overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\therefore$ AX = $\frac{1}{2}$ AB

$\because$ $\overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ $\therefore$ CY = $\frac{1}{2}$ CD

$\because$ AB = CD $\therefore$ AX = CY

$\therefore \Delta\, AXF \text{ , } \Delta\, CYE$

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$

$\therefore \Delta\, AXF \equiv \Delta\, CYE$ then we deduce that $AF = CE$ (Q.E.D. 2)

10

$\because$ Y is the midpoint of $\overline{AC}$ $\quad \therefore \overline{MY} \perp \overline{AC}$ (1)

Similarly $\overline{MX} \perp \overline{AB}$

$\because AC = AB \quad \therefore MY = MX$

and from $\Delta\, YMX$: $\because m(\angle M) = 120^\circ$

$\therefore m(\angle MYX) = m(\angle YXM) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$ (2)

from (1) and (2) : $\therefore m(\angle AYX) = 90^\circ - 30^\circ = 60^\circ$

$\because \overrightarrow{YZ}$ bisects $\angle AYX$

$\therefore m(\angle ZYX) = \frac{60^\circ}{2} = 30^\circ$

$\therefore m(\angle ZYX) = m(\angle YXM)$

but they are alternate angles

$\therefore \overrightarrow{YZ} \parallel \overrightarrow{MX}$ (Q.E.D.)

11

$\because$ The circle M $\cap$ the circle N = {A , B}

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ In $\Delta\, ABD$: $\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

$\therefore AD = BD$

$\because \overline{MX} \perp \overline{AD} \text{ , } \overline{MY} \perp \overline{BD} \quad \therefore MX = MY$ (Q.E.D.)

12

Constr. : Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$, $\overline{MA}$

Proof :

$\because \overleftrightarrow{AB}$ is a tangent to the smaller circle M

$\therefore \overline{MX} \perp \overline{AB}$

, similary : $\overline{MY} \perp \overline{BC}$, $\overline{MZ} \perp \overline{AC}$

, $\because MX = MY = MZ = r$ in the smaller circle

$\therefore AB = BC = AC$

$\therefore \Delta\, ABC$ is an equilateral triangle (First req.)

$\therefore m(\angle B) = 60^\circ$

, $\because$ the greater circle M is the circumcircle of $\Delta\, ABC$

$\therefore$ M is the point of intersection of the altitudes of $\Delta\, ABC$

$\therefore \overline{AY}$ is an altitude in $\Delta\, ABC$

$\therefore$ In $\Delta\, ABY$ which is right at Y : $\sin B = \frac{AY}{AB}$

, $\because AY = AM + MY = 4 + 2 = 6$ cm.

$\therefore \sin 60^\circ = \frac{6}{AB} \qquad \therefore \frac{\sqrt{3}}{2} = \frac{6}{AB}$

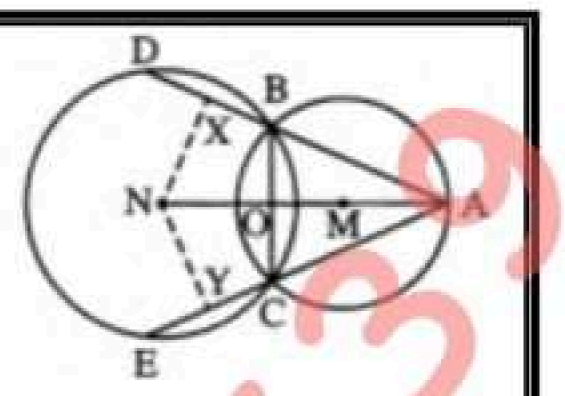
$\therefore AB = \frac{2 \times 6}{\sqrt{3}} = 4\sqrt{3}$ cm. $\quad \therefore BC = AB = 4\sqrt{3}$ cm.

$\therefore$ The area of $\Delta\, ABC = \frac{1}{2} \times BC \times AY$

$= \frac{1}{2} \times 4\sqrt{3} \times 6 = 12\sqrt{3}$ cm^2. (Second req.)

13

Constr. : Draw $\overline{NX} \perp \overline{BD}$, $\overline{NY} \perp \overline{EC}$



Proof :

$\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{BC}$ is the common chord of the two circles

$\therefore \overleftrightarrow{MN} \perp \overline{BC}$, O is the midpoint of $\overline{BC}$

$\therefore OB = OC$

$\therefore$ In $\Delta\Delta$ AOB , AOC

$\begin{cases} OB = OC \\ \overline{AO} \text{ is common side} \\ m(\angle AOB) = m(\angle AOC) = 90^\circ \end{cases}$

$\therefore \Delta\, AOB \equiv \Delta\, AOC$

$\therefore m(\angle BAO) = m(\angle CAO)$

In $\Delta\Delta$ AXN , AYN

$\because m(\angle AXN) = m(\angle AYN) = 90^\circ$

, $m(\angle XAN) = m(\angle YAN)$

$\therefore m(\angle ANX) = m(\angle ANY)$

$\therefore$ In $\Delta\Delta$ AXN , AYN

$\begin{cases} m(\angle ANX) = m(\angle ANY) \\ m(\angle XAN) = m(\angle YAN) \\ \overline{AN} \text{ is a common side} \end{cases}$

$\therefore \Delta\, AXN \equiv \Delta\, AYN \qquad \therefore NX = NY$

, $\because \overline{NX} \perp \overline{BD}$, $\overline{NY} \perp \overline{CE}$

$\therefore BD = CE$ (Q.E.D.)

14

$\because \Delta\, MXA$ and $\Delta\, MYB$ which are right-angled triangles

In them $\begin{cases} MA = MB \text{ (lengths of two radii)} \\ MX = MY \end{cases}$

$\therefore$ The two triangles are congruent , then we deduce that :

$m(\angle MAX) = m(\angle MBY)$

$\therefore \Delta\, HAB$ is an isosceles triangle. (Q.E.D. 1)

, $\because \overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{BD}$

, $MX = MY$

$\therefore AC = BD \qquad , \because AH = BH$

$\therefore AH - AC = BH - BD \quad \therefore HC = HD$ (Q.E.D. 2)

15

$\because$ X is the midpoint of $\overline{AB} \qquad \therefore \overline{MX} \perp \overline{AB}$

$\because$ Y is the midpoint of $\overline{AC} \qquad \therefore \overline{MY} \perp \overline{AC}$

$\because MX = MY \qquad \therefore AB = AC$

, $\because m(\angle BAC) = 60^\circ$

$\therefore \Delta\, ABC$ is an equilateral triangle (Q.E.D. 1)

$\because BM = CM = r \quad \therefore M \in$ the axis of symmetry of $\overline{BC}$

$\because AB = AC \quad \therefore A \in$ the axis of symmetry of $\overline{BC}$

$\therefore \overleftrightarrow{AM}$ is the axis of symmetry of $\overline{BC}$

$\therefore \overleftrightarrow{AM} \perp \overline{BC}$ (Q.E.D. 2)

16

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ similarly $\overline{MY} \perp \overline{CD}$,

$\because AB = CD$ $\qquad \therefore MX = MY$

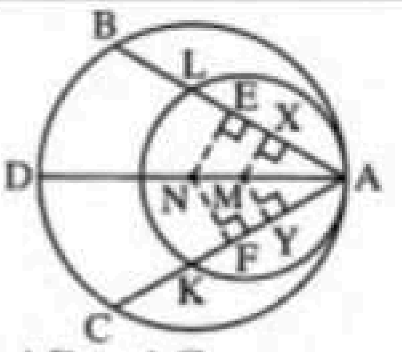
$\therefore \Delta\, MYX$ is an isosceles triangle

$\because \overline{ML} \perp \overline{XY}$ $\qquad \therefore XL = LY$ $\qquad (1)$

$\because \overline{ML} \perp$ the chord $\overline{EF}$ $\qquad \therefore EL = LF$ $\qquad (2)$

subtracting (1) from (2) : $\therefore XE = YF$ (Q.E.D.)

17



Constr. :

Draw : $\overline{NE} \perp \overline{AB}$, $\overline{NF} \perp \overline{AC}$

$\overline{MX} \perp \overline{AL}$, $\overline{MY} \perp \overline{AK}$

Proof : $\because \overline{NE} \perp \overline{AB}$, $\overline{NF} \perp \overline{AC}$, $AB = AC$

$\therefore NE = NF$

$\therefore \Delta\, ANE$ and $\Delta\, ANF$ which are right-angled

In them $\begin{cases} NE = NF \\ \overline{AN} \text{ is a common side} \end{cases}$

$\therefore \Delta\, ANE \equiv \Delta\, ANF$, then we deduce that

$m(\angle NAE) = m(\angle NAF)$

$\therefore \Delta\, AMX$, $\Delta\, AMY$

In them $\begin{cases} \overline{AM} \text{ is common side} \\ m(\angle AXM) = m(\angle AYM) = 90^\circ \\ m(\angle XAM) = m(\angle YAM) \text{ (proved)} \end{cases}$

$\therefore \Delta\, AMX \equiv \Delta\, AMY$, then we deduce that $MX = MY$

$\overline{MX} \perp \overline{AL}$, $\overline{MY} \perp \overline{AK}$

$\therefore AL = AK$ (Q.E.D.)

18

$\because \overline{MD} \perp \overline{AB}$ $\qquad \therefore$ D is the midpoint of $\overline{AB}$ (1)

, $\because \overline{ME} \perp \overline{AC}$ $\qquad \therefore$ E is the midpoint of $\overline{AC}$ (2)

, $\because AD = \sqrt{(2-1)^2 + (2-0)^2} = \sqrt{5}$ length units

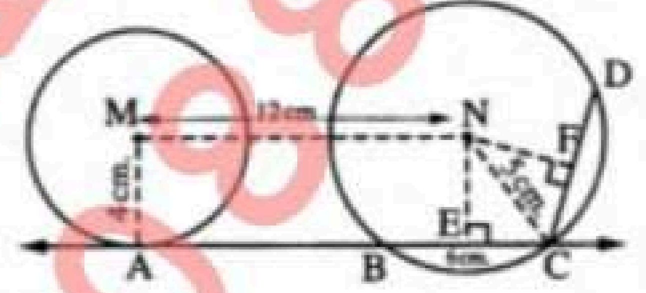
, $AE = \sqrt{(2-3)^2 + (2-4)^2} = \sqrt{5}$ length units

$\therefore AD = AE$ $\qquad \therefore AB = AC$

$\therefore ME = MD$ (Q.E.D.)

19

Constr. :



Draw :

$\overline{NE} \perp \overline{CB}$, $\overline{NF} \perp \overline{CD}$

Proof : $\because \overline{NE} \perp \overline{CB}$

$\therefore$ E is the midpoint of $\overline{CB}$

In $\Delta\, NEC$ which is right-angled at E

$NE = \sqrt{(NC)^2 - (CE)^2} = \sqrt{25 - 9} = 4$ cm.

$\therefore NE = AM$

$\because \overleftrightarrow{AC}$ is a tangent to the circle M , $\overline{MA}$ is a radius

$\therefore \overline{MA} \perp \overline{AC}$

, $\because m(\angle CEN) = m(\angle CAM) = 90^\circ$

and they are alternate angles

$\therefore \overline{NE} \,//\, \overline{AM}$

$\therefore$ The figure NEAM is a rectangle $\qquad \therefore \overline{NM} \,//\, \overline{CA}$

$\therefore$ The figure MACN is a trapezium

Its area $= \frac{1}{2}(MN + AC) \times AM$

$= \frac{1}{2}(12 + 15) \times 4 = 54 \text{ cm}^2$ (First req.)

$\because \overline{NF} \perp \overline{CD}$, $\overline{NE} \perp \overline{CB}$, $CD = CB$

$\therefore NF = NE = 4$ cm.

$\therefore$ The distance between the point N and $\overline{CD}$ is 4 cm.

(Second req.)

RULES OF ALGEBRA

Basic definitions and concepts on the circle

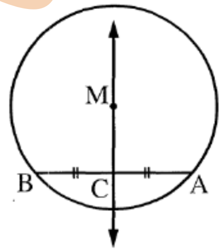
Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M

and C is the midpoint of $\overline{AB}$, then $\overleftrightarrow{MC} \perp \overline{AB}$

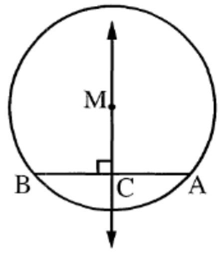


Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then C is the midpoint of $\overline{AB}$



Corollary 3

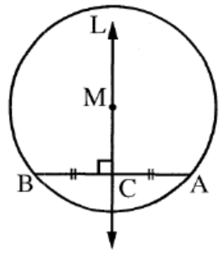
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

and the straight line $L \perp \overline{AB}$ from the point C ,

then $M \in$ the straight line L



From the previous corollary , we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

The radius of the circle

It is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

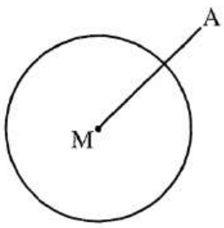
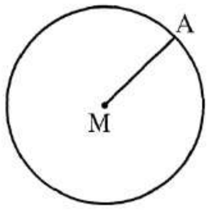
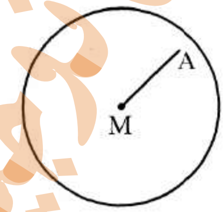
The diameter of the circle

It is a chord passing through the centre of the circle.

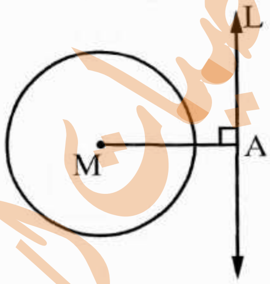
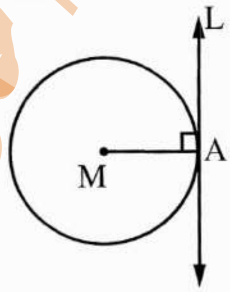
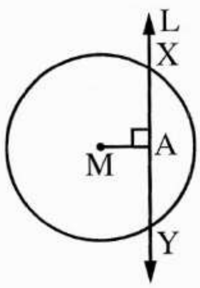
Position of a point and a straight line With respect to a circle

First Position of a point with respect to a given circle :

If M is a circle of radius length r and A is a point in its plane , then :

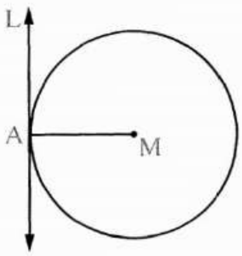
A is outside the circle M	A is on the circle M	A is inside the circle M
		
If MA > r	**If MA = r**	**If MA < r**

Second Position of a straight line with respect to a circle :

If	Then	The figure	Note that
(1) MA > r	The straight line L lies outside the circle M		• L ⋂ the circle M = ∅ • L ⋂ the surface of the circle M = ∅
(2) MA = r	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		• L ⋂ the circle M = {A} • L ⋂ the surface of the circle M = {A}
(3) MA < r	The straight line L is a secant to the circle M		• L ⋂ the circle M = {X , Y} • L ⋂ the surface of the circle M = $\overline{XY}$ $\overline{XY}$ is called the chord of intersection

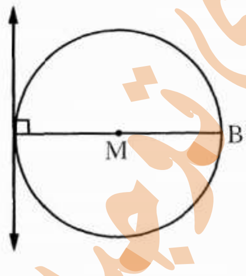
Two important facts

1 **The tangent to a circle is perpendicular to the radius drawn from the point of tangency.**



i.e. if the straight line L is a tangent to the circle M at the point A ,
then $\overline{MA} \perp L$

2 **The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.**

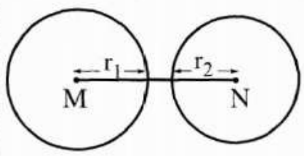
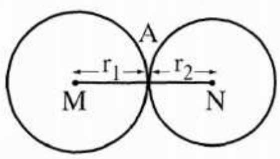
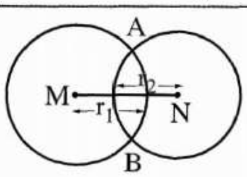


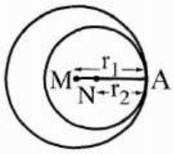
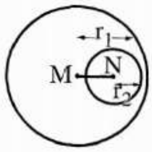
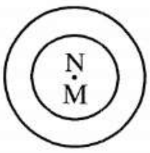
i.e. if $\overline{AB}$ is a diameter of the circle M and the straight line $L \perp \overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

Position of a circle with respect to another circle

Let M and N be two circles , their radii lengths are r_1 and r_2 respectively , $r_1 > r_2$

If	Then the two circles are	Note that
 $MN > r_1 + r_2$	Distant	• The circle M $\cap$ the circle N = $\varnothing$ • The surface of circle M $\cap$ the surface of circle N = $\varnothing$
 $MN = r_1 + r_2$	Touching externally	• The circle M $\cap$ the circle N = {A} • The surface of circle M $\cap$ the surface of circle N = {A}
 $r_1 - r_2 < MN < r_1 + r_2$	Intersecting	• The circle M $\cap$ the circle N = {A , B} • The surface of circle M $\cap$ the surface of circle N = the surface of the yellow part.

 $MN = r_1 - r_2$	Touching internally	• The circle M $\cap$ the circle N = {A} • The surface of circle M $\cap$ the surface of circle N = the surface of circle N
 $MN < r_1 - r_2$	One inside the other the circle N is inside the circle M	• The circle M $\cap$ the circle N = $\varnothing$ • The surface of circle M $\cap$ the surface of circle N = the surface of circle N
 MN = zero	Concentric	• The circle M $\cap$ the circle N = $\varnothing$ • The surface of circle M $\cap$ the surface of circle N = the surface of circle N

Summary

0		$r_1 - r_2$		$r_1 + r_2$		The length of $\overline{MN}$
Concentric	One inside the other	**Touching internally**	Intersecting	**Touching externally**	Distant	

Notes :

From the previous summary , we notice that :

1 If M and N are two distant circles , then : $MN \in]\, r_1 + r_2 , \infty[$

2 If M and N are two intersecting circles , then : $MN \in]\, r_1 - r_2 , r_1 + r_2[$

3 If M and N (one of them is inside the other) , then : $MN \in]\, 0 , r_1 - r_2[$

Corollary 1

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

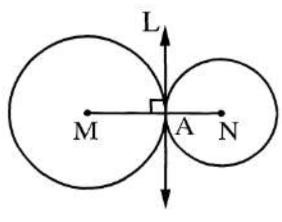
If the two circles

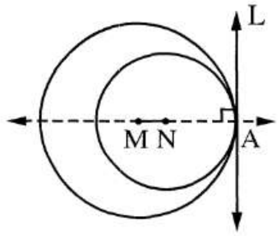
M and N are touching

at A (the point of tangency)

the straight line L is a common tangent to them at A

then $A \in \overleftrightarrow{MN}$ and $\overleftrightarrow{MN} \perp$ the straight line L



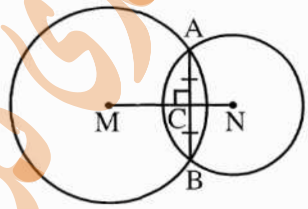


Corollary 2

The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,
then $\overleftrightarrow{MN} \perp \overline{AB}$, $\overleftrightarrow{MN}$ bisects $\overline{AB}$ i.e. AC = BC
This mean that $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$



LESSON [4] Identifying the circle

We know that the circle is identified if we know :

1 its centre **2** its radius length

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

First Drawing a circle passing through a given point :

i.e. **We can draw an infinite number of circles passing through a given point.**

Second Drawing a circle passing through two given points :

i.e. **There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$**

Remarks

If $\overline{AB}$ is a line segment and the required is drawing a circle passing through the two points A and B , then :

1 If $\boxed{r > \frac{1}{2} AB}$, then we can draw two circles (as shown in the previous example).

2 If $\boxed{r = \frac{1}{2} AB}$, then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence $\overline{AB}$ is a diameter of it and its centre is the midpoint of $\overline{AB}$

3 If $\boxed{r < \frac{1}{2} AB}$, then it is impossible to draw any circle.

- **Any two circles do not intersect at more than two points.**

Third Drawing a circle passing through three given points :

i.e. **It is impossible to draw a circle passing through three collinear points.**

For any three non-collinear points , there is a unique circle

can be drawn to pass through them.

Notice that :

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

Corollary 1

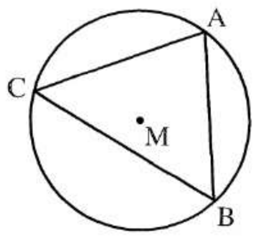
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

M is the circumcircle of Δ ABC

or Δ ABC is the inscribed triangle of the circle M

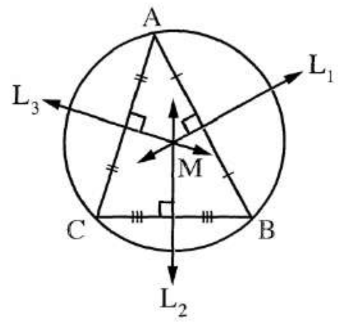


Corollary 2

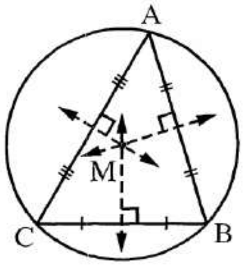
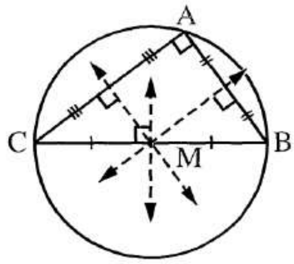
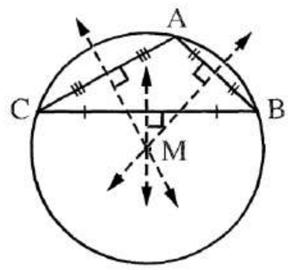
The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

In the opposite figure :

If the straight lines L_1 , L_2 and L_3 are the axes of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively and $L_1 \cap L_2 \cap L_3 = \{M\}$, then the point M is the centre of the circumcircle of Δ ABC



The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :

The acute-angled triangle	The right-angled triangle	The obtuse-angled triangle
 M is **inside** the triangle	 M is the **midpoint of the hypotenuse**	 M is **outside** the triangle

A special case :

The centre of the circumcircle of an equilateral triangle is :

- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its angles.

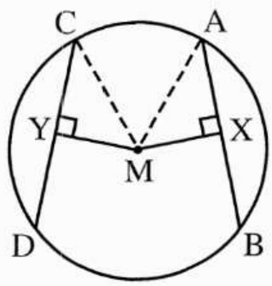
Notice that :

We can draw a circle passing through the vertices of (a rectangle or a square or an isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram or the rhombus or the trapezium which is not isosceles).

The relation between the chords of a circle and its center

Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.



Given	AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$
R.T.P.	MX = MY
Construction	Draw $\overline{MA}$ and $\overline{MC}$

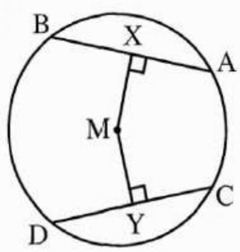
Corollary

In congruent circles, chords which are equal in length are equidistant from the centres.

Converse of the theorem (without proof) :

In the same circle (or in congruent circles) ,
chords which are equidistant from the centre (s) are equal in length.

***i.e.* In the opposite figure :**



If $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ and MX = MY , then AB = CD

Central angles and measuring arcs

i.e.

The measure of the semicircle = 180° and then
the measure of the circle = 2 × 180° = 360°

Remark

The two adjacent arcs are two arcs in the same circle that have only one point in common.

The length of the arc

It is a part of a circle's circumference proportional to its measure and it is measured by length units (centimetre , metre , ...)

- To calculate the length of the arc, you can use the following rule :

$$\text{The length of the arc} = \frac{\text{the measure of the arc}}{\text{the measure of the circle}} \times \text{the circumference of the circle}$$

$$= \frac{\text{the measure of the arc}}{360^\circ} \times 2\,\pi\,r$$

Where r is the radius length of the circle and π is the approximated ratio.

Corollary 1

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.

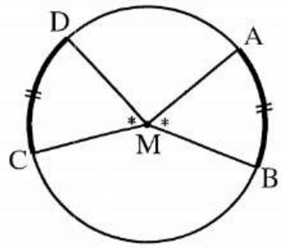
In the opposite figure :

If M is a circle in which $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

and vice versa if the length of $\overset{\frown}{AB}$

= the length of $\overset{\frown}{CD}$, then $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$



Corollary 2

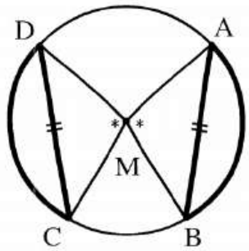
In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.

In the opposite figure :

If M is a circle in which

$m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$, then AB = CD and vice versa

If AB = CD , then $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$



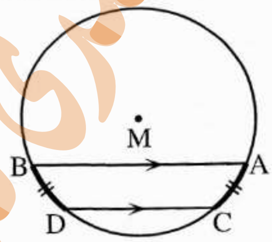
Corollary 3

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{AB} \parallel \overline{CD}$, then m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BD})$



Corollary 4

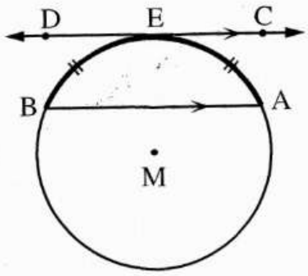
If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If $\overline{AB}$ is a chord in the circle M and

$\overleftrightarrow{CD}$ touches the circle M at E ,

$\overleftrightarrow{CD} \parallel \overline{AB}$, then m $(\overset{\frown}{EA})$ = m $(\overset{\frown}{EB})$



The relation between the inscribed and Central angles subtended by the same arc theorem

The inscribed angle

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

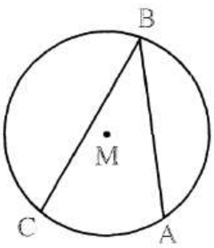
In the opposite figure :

- $\angle$ ABC is an inscribed angle

  because its vertex B belongs to the circle M

  and its sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ carry the two chords $\overline{BA}$ and $\overline{BC}$ in the circle M

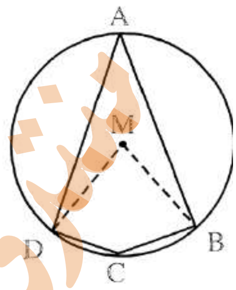
- The inscribed angle $\angle$ ABC is subtended by $\overset{\frown}{AC}$



Remark

For each inscribed angle , there is one central angle subtended by the same arc.

In the opposite figure :



- The inscribed angle $\angle$ BAD is subtended

  with the central angle $\angle$ BMD by the arc $\overset{\frown}{BCD}$

- While the inscribed angle $\angle$ BCD is subtended

  with the reflex central angle BMD by the arc $\overset{\frown}{BAD}$

Theorem (1)

The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.

Remark

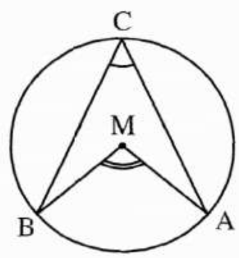
The measure of the central angle equals twice the measure of the inscribed angle subtended by the same arc.

The corollaries of theorem (1) and its well known problems

Corollary 1

The measure of an inscribed angle is half the measure of the subtended arc.

In the opposite figure :



m ($\angle$ C) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles with common arc $\overset{\frown}{AB}$) ,

m ($\angle$ AMB) = m ($\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ C) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$)

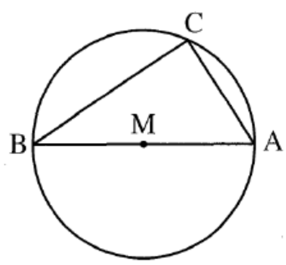
Remark

The measure of the arc equals twice the measure of the inscribed angle subtended by this arc.

Corollary 2

The inscribed angle in a semicircle is a right angle.

In the opposite figure :



$\because$ m ($\angle$ C) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) (corollary 1) ,

$\therefore$ m ($\overset{\frown}{AB}$) = 180°

$\therefore$ m ($\angle$ C) = 90°

Remarks

1 The inscribed angle which is right angle is drawn in a semicircle.

2 The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.

3 The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

Well known problem (1)

If two chords intersect at a point inside a circle , then the measure of the included angle equals half of the sum of the two measures of the two opposite arcs.

Given $\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E

R.T.P.

1 $m(\angle AEC) = \frac{1}{2}\left[m(\widehat{AC}) + m(\widehat{BD})\right]$

2 $m(\angle CEB) = \frac{1}{2}\left[m(\widehat{BC}) + m(\widehat{AD})\right]$

Well known problem (2)

If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.

Given $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

R.T.P. $m(\angle A) = \frac{1}{2}\left[m(\widehat{CE}) - m(\widehat{BD})\right]$

Inscribed angles subtended by same arc

Theorem (2) its corollaries

Theorem 2

In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

Given $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\widehat{AB}$

R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$

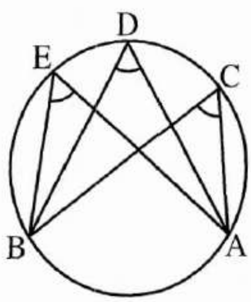
Proof

$\because m(\angle C) = \frac{1}{2} m(\widehat{AB})$

, $m(\angle D) = \frac{1}{2} m(\widehat{AB})$

, $m(\angle E) = \frac{1}{2} m(\widehat{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$ (Q.E.D.)



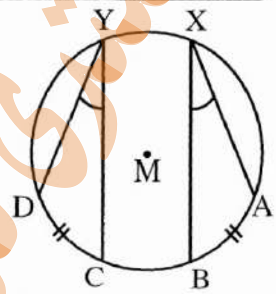
Corollary

In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.

i.e. In the circle M

If m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$,

then m (∠ X) = m (∠ Y)



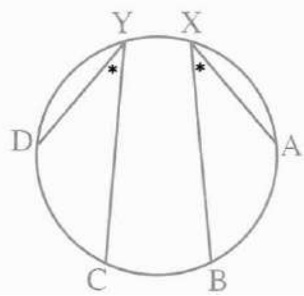
The converse of the previous corollary is true also

i.e. In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.

In the opposite figure :

If m (∠ X) = m (∠ Y) ,

then m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$



The cyclic quadrilateral-the converse of theorem (2)

The cyclic quadrilateral :

It is a quadrilateral figure whose four vertices belong to one circle.

The converse of theorem (2) (without proof)

If two angles subtended by the same base and on the same side of it have the same measure , then their vertices are on an arc of a circle and the base is a chord of it.

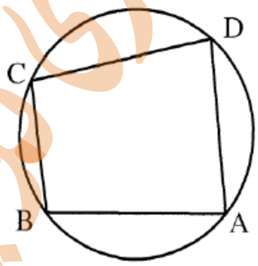
Remarks

1. If there are two angles drawn on one of the sides of a quadrilateral , they are on the same side of it and they are not equal in measure , then the quadrilateral is not cyclic.
2. Each of the rectangle , the square and the isosceles trapezium are cyclic quadrilaterals while each of the parallelogram , the rhombus and the trapezium that is not isosceles are not cyclic quadrilaterals.

Properties of The cyclic quadrilateral theorem (3)

Theorem 3

In a cyclic quadrilateral , each two opposite angles are supplementary.



Given ABCD is a cyclic quadrilateral

R.T.P.

1 $m(\angle A) + m(\angle C) = 180°$

2 $m(\angle B) + m(\angle D) = 180°$

Proof

$\because m(\angle A) = \frac{1}{2}\, m(\overset{\frown}{BCD})$ and $m(\angle C) = \frac{1}{2}\, m(\overset{\frown}{BAD})$

$\because m(\angle A) + m(\angle C) = \frac{1}{2}\left[m(\overset{\frown}{BCD}) + m(\overset{\frown}{BAD})\right]$

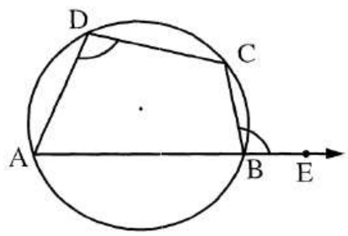
$= \frac{1}{2}$ the measure of the circle $= \frac{1}{2} \times 360° = 180°$

Similarly: $m(\angle B) + m(\angle D) = 180°$ (Q.E.D.)

Corollary

The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.

In the opposite figure :



If ABCD is a cyclic quadrilateral

, $\angle CBE$ is an exterior angle of it ,

then $m(\angle ABC) + m(\angle D) = 180°$

but $m(\angle ABC) + m(\angle CBE) = 180°$

$\therefore m(\angle CBE) = m(\angle D)$

The converse of theorem (3) and its corollary

A summary of the cases in which the quadrilateral is cyclic :

The quadrilateral is cyclic if one of the following conditions is verified :

1 If there is a point in the plane of the figure such that it is equidistant from its vertices.

2 If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.

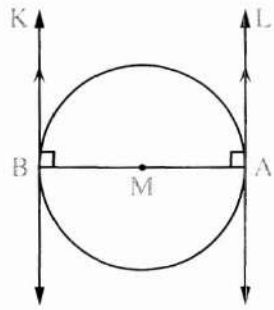
3 If there are two opposite supplementary angles «their sum = 180°»

4 If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.

The relation between the tangents of a Circle theorem (4) and its corollaries

First | **The two tangents drawn at the two ends of a diameter in a circle are parallel.**

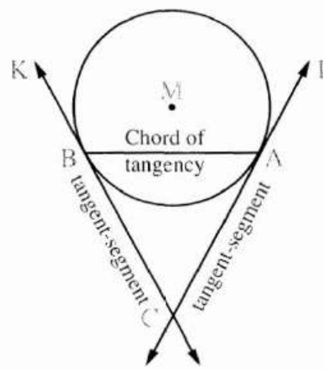
i.e. In the opposite figure :



If $\overline{AB}$ is a diameter in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively,
then the straight line L // the straight line K
(because the straight line L $\perp$ $\overline{AB}$ and the straight line K $\perp$ $\overline{AB}$)

Second | **The two tangents drawn at the two ends of a chord of a circle are intersecting.**

i.e. In the opposite figure :



If $\overline{AB}$ is a chord in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively, then the two straight lines L and K are intersecting at a point outside the circle M (Say C) and $\overline{AC}$, $\overline{BC}$ are called tangent - segments and $\overline{AB}$ is called a chord of tangency.

Theorem (4)

The two tangent-segments drawn to a circle from a point outside it are equal in length.

Corollaries of theorem (4) :

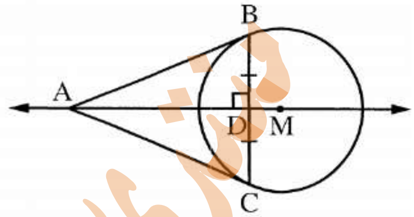
Corollary 1

The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.

In the opposite figure :

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively, then $\overleftrightarrow{AM}$ is the axis of symmetry to $\overline{BC}$

i.e. $\overleftrightarrow{AM} \perp \overline{BC}$, BD = CD



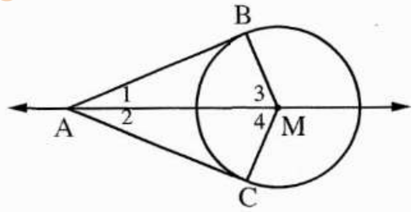
Corollary 2

The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.

In the opposite figure :

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively then :

- $\overrightarrow{AM}$ bisects $\angle$ BAC $\quad \therefore$ m ($\angle$ 1) = m ($\angle$ 2)
- $\overrightarrow{MA}$ bisects $\angle$ BMC $\quad \therefore$ m ($\angle$ 3) = m ($\angle$ 4)



Remarks on theorem (4) and its corollaries

In the opposite figure :

1 AB = AC

2 MB = MC = r

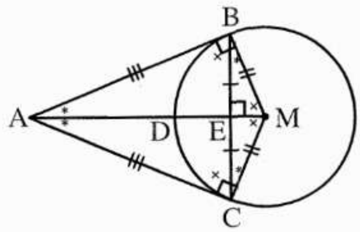
3 BE = CE , $\overleftrightarrow{AM} \perp \overline{BC}$

4 m ($\angle$ ABM) = m ($\angle$ ACM) = 90°

i.e. The figure ABMC is a cyclic quadrilateral.

5 m ($\angle$ BAM) = m ($\angle$ BCM) = m ($\angle$ CAM) = m ($\angle$ CBM)

6 m ($\angle$ AMB) = m ($\angle$ ACB) = m ($\angle$ AMC) = m ($\angle$ ABC)



Definition

The inscribed circle of a polygon is the circle which touches all of its sides internally.

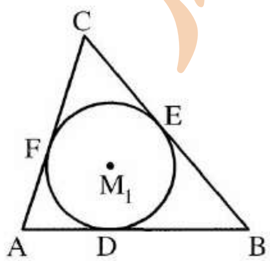


Fig. (1)

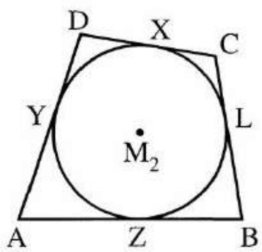


Fig. (2)

In figure (1) : M_1 is the inscribed circle of the triangle ABC where :

the side $\overline{AB}$ touches the circle at D , the side $\overline{BC}$ touches the circle at E and the side $\overline{CA}$ touches the circle at F.

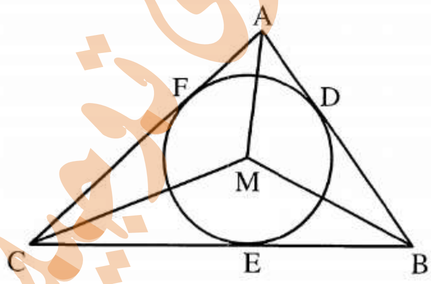
In figure (2) : M_2 is the inscribed circle of the quadrilateral ABCD

Remark

The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.

i.e. In the opposite figure :

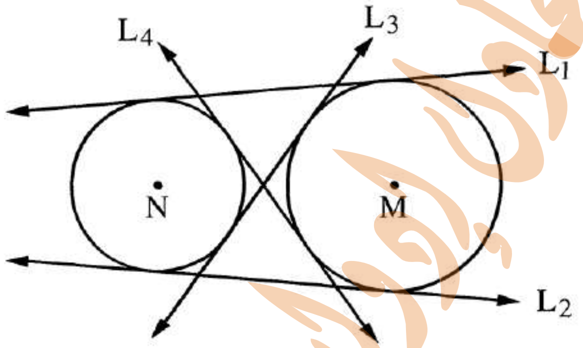
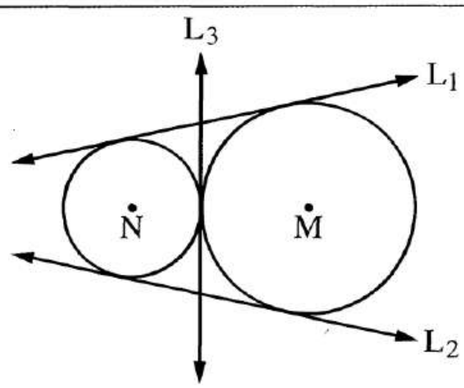
If the circle M is the inscribed circle of the triangle ABC then M is the intersection point of the bisectors of the interior angles of Δ ABC

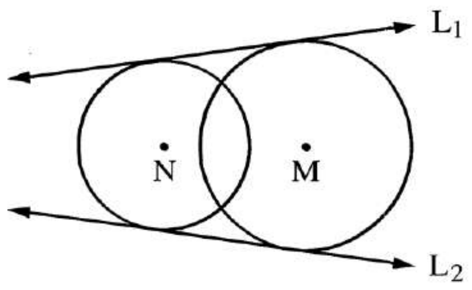
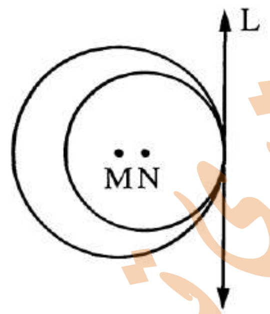


The common tangents to two circles

- It is said that the tangent $\overleftrightarrow{AB}$ is an internal common tangent to the two circles M and N if the two circles M and N are on two different sides of the tangent.
- It is said that the tangent $\overleftrightarrow{AB}$ is an external common tangent of the two circles M and N if the two circles M and N are on the same side of the tangent.

The following table shows the number of the common tangents to two circles in their different situations (locations) :

Two distant circles	Two circles touching externally
 **4 common tangents** • L_1 and L_2 (external) • L_3 and L_4 (internal)	 **3 common tangents** • L_1 and L_2 (external) • L_3 (internal)

Two intersecting circles	Two circles touching internally
 **2 common tangents** • L_1 and L_2 (external) • There are no internal tangents	 **One common tangent** • L is the common tangent (external) • There are no internal tangents

One circle inside the other
 **There are no common tangents**

Angles of tangency theorem (5) and its corollaries

Definition

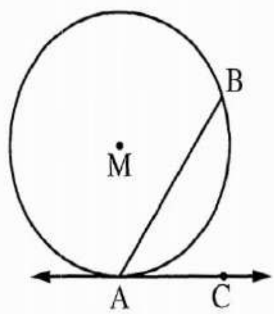
The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.

In the opposite figure :

If $\overleftrightarrow{AC}$ is a tangent to the circle at A and $\overrightarrow{AB}$ contains the chord $\overline{AB}$

, then $\angle$ BAC is an angle of tangency in the circle M , its chord is $\overline{AB}$

$\overline{AB}$ is called the chord of tangency of the angle of tangency $\angle$ BAC



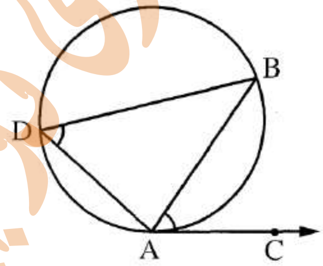
i.e. **The measure of the angle of tangency = $\frac{1}{2}$ the measure of the arc intercepted by its sides.**

Theorem 5

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

Given $\angle$ BAC is an angle of tangency and $\angle$ D is an inscribed angle.

R.T.P. m ($\angle$ BAC) = m ($\angle$ D)



Proof $\because$ $\angle$ BAC is an angle of tangency.

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\widehat{AB}$) (1)

, $\because$ $\angle$ D is an inscribed angle

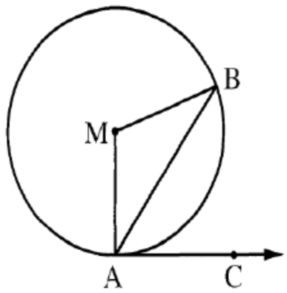
$\therefore$ m ($\angle$ D) = $\frac{1}{2}$ m ($\widehat{AB}$) (2)

From (1) and (2) , we deduce that : m ($\angle$ BAC) = m ($\angle$ D) (Q.E.D.)

Corollary

The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.

In the opposite figure :



m ($\angle$ BAC) (tangency angle) = $\frac{1}{2}$ m ($\widehat{AB}$)

, $\because$ m ($\angle$ AMB) (central angle) = m ($\widehat{AB}$)

$\therefore$ m ($\angle$ BAC) (tangency angle) = $\frac{1}{2}$ m ($\angle$ AMB) (central angle)

Remark

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

The converse of theorem (5)

If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side , then this ray is a tangent to the circle.

Questions Part (1)

Geometry

General Exercise on Unit One

First: Complete the following

1) If one end of a line segment lies on the center of the circle and the other end on the circle, then this line segment is called ....
2) If the two ends of a line segment lie on the circle, then this line segment is called ....
3) The chord which passes through the center of the circle is called ....
4) The longest chord of the circle is called ....
5) The circle has .... number of axes of symmetry.
6) In any circle the perpendicular straight line on any chord from its mid-point is .... to the circle.
7) The circle divides the plane into .... sets of points.
8) The perpendicular straight line on the diameter from one end is ....
9) The two tangents to a circle at the two end points of the diameter are ....
10) The equal chords in length of a circle are equidistant from ....
11) The chords of a circle are equidistant from its center are ....
12) If the point A lies outside the circle M of radius, then MA .... R.
13) The line of centers of two intersecting circles is .... , ...
14) If the surface of the circle M $\cap$ the surface of the circle N = φ , then the two circles M and N are.......
15) If the surface of the circle M $\cap$ the surface of the circle N = {A}, then the two circles M and N are ....
16) The number of circles can be drawn passing through two given points in the plane equals ....
17) If two circles have three common points, then they are ....
18) The radius of the smallest circle drawn to pass through two given points in the plane equals ....
19) The point of intersection of the symmetric axes of the sides of a triangle is ....
20) If M is a circle of radius r, A is apoint in the plane of the circle:
    (a) If MA = $\frac{1}{2}$ R, then A .... the circle
    (b) If MA = R, then A .... the circle
    (c) If MA = 3 R, then A .... the circle

Second: Match from the column (X) to the column (Y) to get a true statement

Two circles of radii 8 cm. & 6 cm.

X	Y
1) If MN = 1 cm	a) M , N are two intersecting circles
2) If MN = 2 cm	b) M , N are two distant circles
3) If MN = 7 cm	c) M , N touching externally
4) If MN = 14 cm	d) M , N are two interior circles
5) If MN = 15 cm	e) M , N touching internally

Third : Choose the correct from the given ones :

1) If the length of a diameter of a circle is 7 cm, and the straight line L at distant 3.5 cm from its center, then L is ....
   a) Secant to the circle at two points
   b) Lies outside the circle.
   c) Tangent to the circle
   d) Axis of symmetry to the circle
2) If the point A belongs to the circle M of diameter 6 cm, then MA equals
   a) 3 cm
   b) 4 cm
   c) 5 cm
   d) 6 cm
3) If the straight line L is a tangent to the circle M of diameter 8 cm, then the distance between L and its center equals ....
   a) 3 cm
   b) 4 cm
   c) 6 cm
   d) 8 cm
4) If the straight line L is outside a circle of radius 3 cm and its center M, If L at distance X from its center, then x ∈ ....
   a)]3 , ∞[
   b) [3 , ∞ [
   c)]6 , ∞ [
   d)]- ∞ , - 6[
5) If the straight line L at distance x from a circle of center M and radius R, x ∈]0,R[, then L ....
   a) Intersects the circle
   b) Touches the circle
   c) Lies outside the circle
   d) Passes through the center of the circel
6) If the length of the perpendicular drawn from the center of the circle on the straight line L equals 6 cm and the radius 6 cm, then L ....
   a) Intersects the circle
   b) Touches the circle
   c) Lies outside the circle
   d) Passes through the center of the circle

7) Which of the following points does not belong to the circle that its center is the origin and its radius 7 cm?

a) (0 , 7) b) (0 , - 7) c) (7 , 0) d) (7 , 7)

8) If the surface of the circle M ∩ the surface of the circle N = {A}, then the two circles M and N are.......

a) Distant b) Concentric

c) Touching externally d) Intersecting

9) The number of circles can be drawn to pass through the end points of the line segment $\overline{AB}$ equals ......

a) 1 b) 2 c) 3 d) an infinite number

10) If the circle M ∩ the circle N = {A , B}, then the two circles M and N are ....

a) Distant b) Concentric

c) Touching externally d) Intersecting

11) If the two circles M, N are touching externally, the radius of one of them 5 cm, and MN = 9 cm, then the radius of the other circle equals ....

a) 3 cm b) 4 cm c) 7 cm d) 14 cm

12) If the two circles M, N are touching internally, the radius of one of them 3 cm, and MN = 8 cm, then the radius of the other circle equals ....

a) 5 cm b) 6 cm c) 11 cm d) 12 cm

13) M and N are two intersecting circles their radii are 5 cm, 2 cm, then MN = ...

a)]3 , 7[b) [3 , 7[c)]3 , 7] d) [3 , 7]

14) The number of circles that pass through three collinear points equals ....

a) zero b) One c) Three d) An infinte number

15) The symmetric axis of the common chord $\overline{AB}$ to the two intersecting circles M, N is ....

a) $\overleftrightarrow{MA}$ b) $\overleftrightarrow{MB}$ c) $\overleftrightarrow{MN}$ d) $\overleftrightarrow{NA}$

16) The centers of the circles which pass through the two points A, B lie on ....

a) The axis of $\overline{AB}$ b) $\overline{AB}$

c) The perpendicular to $\overline{AB}$ d) The perpendicular on $\overline{AB}$ at B

17) Number of the circles which pass through three non collinear points equals ....

a) zero b) one c) two d) three

18) The center of the circumcircle of any triangle is the point of intersection of its ....

a) Interior bisectors of its angles b) Exterior bisectors of its angles

c) its heights d) The symmetric axis of its sides

19) If the two points A, B lie on a plane AB = 4 cm, then the length of the radius of the smallest circle passes through A and B equals ....

a) 2cm b) 3cm c) 4cm d) 8cm

20) If the two points A, B lie on a plane, AB = 6 cm, then the number of circles each of them has a radius of 5 cm and passes through A and B equals ....

a) zero b) 1 c) 2 d) an infinite number

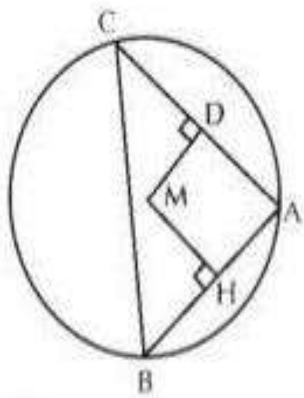
Fourth: Answer the following questions

1) In the opposite figure

ABC is a triangle in a circle of center M,

$\overline{MD} \perp \overline{AC}$, $\overline{ME} \perp \overline{AB}$ and

BC = 8 cm Find DH



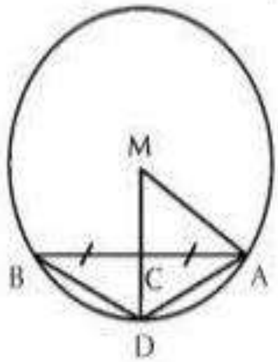
2) In the opposite figure

M circle of center M, its radius of length 13 cm,

$\overline{AB}$ is a chord of length 24 cm, C is the mid point

of $\overline{AB}$, $\overline{MC}$ cuts the circle at D. Find :

[1] The length of $\overline{MC}$ [2] the area of Δ ADB

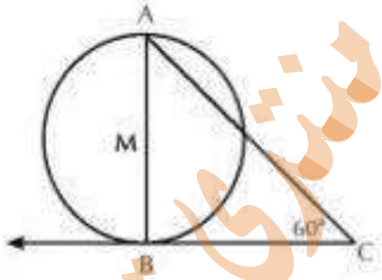


3) In the opposite figure

A circle of circumference 44 cm, $\overline{AB}$ is a diameter

$\overleftrightarrow{BC}$ is a tangent at B, and m ($\angle$C) = 60°.

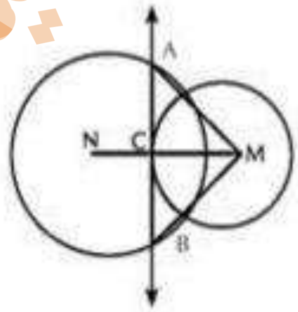
Find the length of $\overline{BC}$ $\quad (\pi = \frac{22}{7})$



4) In the opposite figure

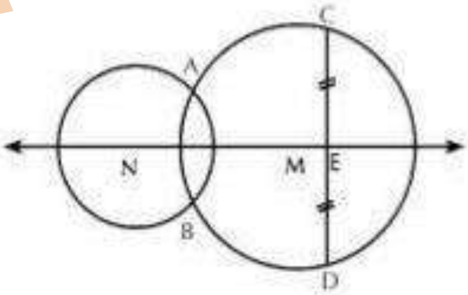
M , N are two intersecting circles, $\overline{MN}$ intersects the circle M at C, $\overline{CA}$ is a tangent to the circle M at C, and cuts the circle N at A, B Prove that :

[1] CA = CB [2] MA = MB



5) In the opposite figure

M, N are two intersecting circles, $\overline{CD}$ is a chord in the circle M, cuts $\overleftrightarrow{MN}$ at E, if E is the mid point of $\overline{CD}$ Prove that : $\overline{AB}$ // $\overline{CD}$



6) M , N are two touching internally circles at A, the circle M is greater than the circle N, draw the common tangent $\overleftrightarrow{AC}$, then draw $\overleftrightarrow{NM}$ to cut the circle N at B, and draw the tangent $\overleftrightarrow{BD}$ to the circle N to cut the circle M at D, E Prove that:

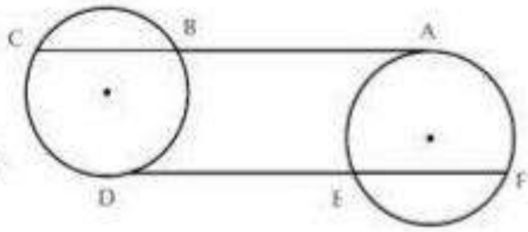
[1] $\overleftrightarrow{AC}$ // $\overleftrightarrow{BD}$

[2] BD = BE.

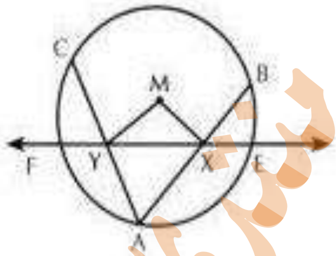
7) In the opposite figure

M , N are two congruent circles, $\overline{AC}$ is a common tangent to the circle M at A the, $\overline{DF}$ is a common tangent to the circle N at D, $\overline{AC}$ // $\overline{DF}$. Prove that :

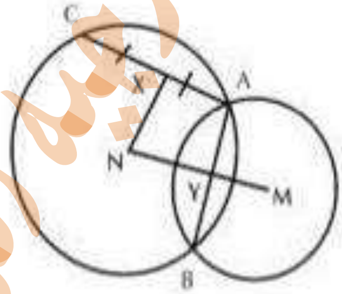
[1] BC = FE [2] AB = ED



8) In the opposite figure
$\overline{AB}$, $\overline{CD}$ are two chords (equal in length) in a circle M.
If X, Y are the two mid points of $\overline{AB}$, $\overline{AC}$ respectively,
$\overleftrightarrow{XY}$ cuts the circle at E and F. Prove that :
XE = YF.



9) In the opposite figure
M , N are two intersecting circles at A, B,
$\overleftrightarrow{MN} \cap \overline{AB} = \{Y\}$, AB = AC, if
X is the mid point of $\overline{AC}$.
Prove that : NX = NY.



10) $\overline{AB}$, $\overline{CD}$ are two parallel chords in a circle M, E is the midpoint of $\overline{AB}$, $\overrightarrow{EM}$ is drawn to cut $\overline{CD}$ at F. Prove that : FC = FD.

11) $\overline{AB}$, $\overline{AC}$ are two chords in a circle M, if D, E are the two the mid points of $\overline{AB}$, $\overline{AC}$ respectively, $\overrightarrow{DM}$ is drawn to cut $\overleftrightarrow{AC}$ at F such ME = EF. Prove that: m (∠BAC) = 45°.

12) $\overline{AB}$ is a diameter in a circle M, the chord $\overline{CD}$ is drawn such that $\overline{CD}$ // $\overline{AB}$, $\overline{CX} \perp \overline{AB}$ and $\overline{DY} \perp \overline{AB}$ Prove that : AX = BY.

13) A, B are two points wher AB = 6 cm, Draw a circle of radius 5 cm and passes through the two points A, B. Find the distance from the center to $\overline{AB}$.

14) Draw the triangle ABC in which AB = 6 cm, AC = 4 cm , m (∠BAC) = 60° . Then draw a circle passes through the two points A, C and its center $\in \overline{AB}$.

15) $\overline{AB}$ is a diameter in a circle M, $\overline{AC}$ is a chord such that m (∠BAC) = 30°, then draw $\overline{BC}$ and $\overline{MD} \perp \overline{AC}$ to cut it at D. Prove that :
[1] $\overline{MD}$ // $\overline{BC}$
[2] BC = the length of the radius of this circle.

General Exercise on the Second Unit

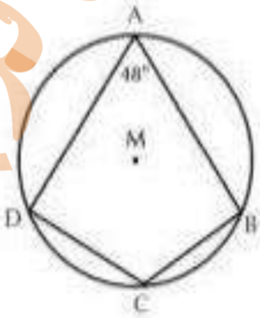
First : Complete the following

(1) The two opposite angles in the cyclic quadrilateral are ....

(2) The chords which opposite to equal arcs in any circle are ....

(3) The measure of the inscribed angle equals half the measure of ....

(4) In the opposite figure



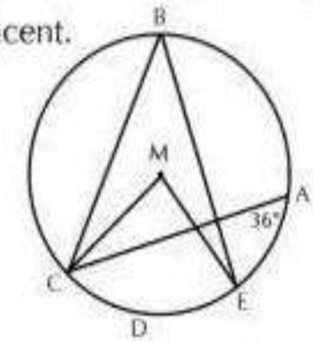
In a circle M, $m(\angle A) = 48°$, then:

[1] $m(\angle C) =$ ....

[2] $m(\overset{\frown}{BD}) =$ .... " $\overset{\frown}{BD}$ is the major arc"

(5) The quadrilateral is said to be a cyclic quad. If the measure of an exterior angle at any vertex equals the .... of the angle which opposite to its adjacent.

(6) In the opposite figure



In a circle M, $m(\angle CAE) = 36°$, then:

(a) $m(\angle EBC) =$ .... (b) $m(\angle EMC) =$ ......

(c) $m(\angle EDC) =$ ....

(7) The inscribed angle which opposite to a minor arc in a circle is ....

(8) The two parallel chords in a circle intercept two .... arcs

(9) The measure of an arc of a circle equals double ....

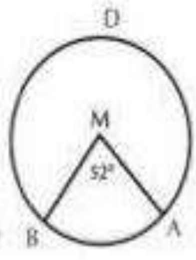
Second : Choose the correct answer from the given ones

1) The inscribed angle which opposite to the minor arc in a circle is ....

(a) reflex (b) right

(c) obtuse (d) acute

2) In the opposite figure



In a circle M, $m(\angle AMB) = 52°$, then

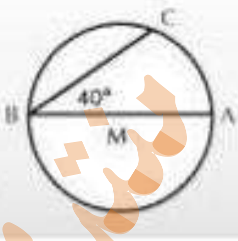
$m(\angle ADB) =$ .... °

(a) 52° (b) 104° (c) 128° (d) 308°

3) In the opposite figure

$\overline{AB}$ is a diameter in a circle M,

m(∠ABC) = 40°, then m($\overset{\frown}{BC}$) = ...°

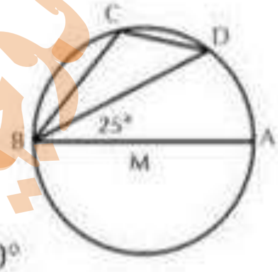


(a) 40° (b) 50° (c) 90° (d) 100°

4) In the opposite figure

$\overline{AB}$ is a diameter in a circle M,

m(∠ABD) = 25°, then



[1] m(∠DAB) = ...°

(a) 25° (b) 50° (c) 65° (d) 90°

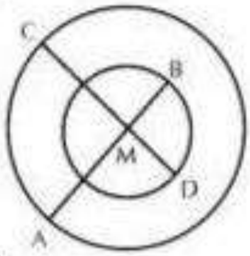
[2] m(∠DCB) = ...°

(a) 50° (b) 100° (c) 115° (d) 125°

5) In the opposite figure

Two concentric circles at M, $\overline{AB} \cap \overline{CD}$ = {M},

if m($\overset{\frown}{BD}$) = 80°, then m($\overset{\frown}{AC}$) = ...°



(a) 40° (b) 80° (c) 100° (d) 160°

6) Using the following figures choose the correct answer

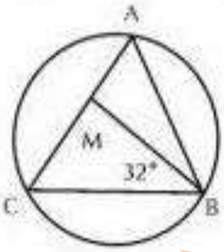


Figure (1)

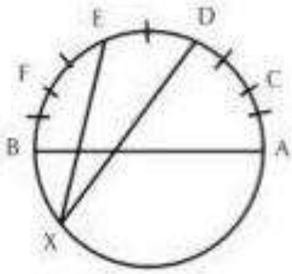


Figure (2)

In Figure (1) : A circle of center M, m(∠MBC) = 32°, then m($\overset{\frown}{BC}$) = ...°

(a) 16° (b) 32° (c) 64° (d) 116°

In Figure (2) : $\overline{AB}$ is a diameter in a circle,

m($\overset{\frown}{AC}$) = m($\overset{\frown}{CD}$) = m($\overset{\frown}{DE}$) = m($\overset{\frown}{EF}$) = m($\overset{\frown}{FB}$), then m(∠DXE) = ...°

(a) 18° (b) 36° (c) 54° (d) 72°

Model Answers Part (1)

First Complete:

(1) radius (2) chord (3) diameter (4) diameter

(5) infinite (6) axis of symmetry (7) three

(8) tangent (9) parallel (10) its centre

(11) equal in length (12) >

(13) perpendicular to the common chord and bisect it

(14) distant (15) touching externally

(16) infinite (17) congruent (coincide)

(18) half the length of the line segment joining the two points.

(19) the centre of the circumcircle.

(20) inside – lies on – outside.

Second match:

(1) (d) (2) (e) (3) (a) (4) (c) (5) (b)

Third choose :

(1) c	(2) a	(3) b	(4) a	(5) a
(6) b	(7) d	(8) c	(9) d	(10) d
(11) b	(12) c	(13) c	(14) a	(15) c
(16) a	(17) b	(18) d	(19) a	(20) c

Fourth Answer the following questions:

(1) $\because \overline{MD} \perp \overline{AC}$, $\because \overline{MH} \perp \overline{AB}$

$\therefore$ D , H are mid points of $\overline{AC}$ and $\overline{AB}$ respectively

$\therefore$ DH = $\frac{1}{2}$CB = 8 ÷ 2 = 4 cm.

(2) $\because$ C is a mid point of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$

In Δ AMC

$MC^2 = AM^2 - AC^2 = 169 - 144 = 25$

MC = 5 cm , $\because$ MD = 13 cm

$\therefore$ CD = 13 – 5 = 8 cm.

area of Δ ADB $= \frac{1}{2}$ x AB x DC

$= \frac{1}{2}$ x 24 x 8 = 96cm^2

(3) cir. $= \pi$ XD $\Longrightarrow 44 = \frac{22}{7}$ x D

$\therefore$ D = 14 cm

$\because \overleftrightarrow{BC}$ is a tangent.

$\therefore m(\angle ABC) = 90°$, $\because m(\angle A) = 30°$

$\therefore BC = \frac{1}{2} AC$

Let BC = x , AC = 2x

$AC^2 = AB^2 + BC^2$

$(2x)^2 = (14)^2 + (x)^2$

$4x^2 = 196 + x^2$

$3x^2 = 196 \longrightarrow x^2 \simeq 65.33$

$\therefore x = \sqrt{65.33} \simeq 8.08$ cm = BC

(4) $\because \overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents

$\therefore \overline{MA} \perp \overline{AC}$, $\overline{MB} \perp \overline{BC}$

$\because m(\angle A) + m(\angle B) = 180°$

$\therefore$ AMBC is acyclic quad.

$\therefore m(\angle DMB) = m(\angle ACB)$

Exterior = opposite interior

(4) In circle M $\because \overleftrightarrow{AC}$ is a tangent

$\therefore \overline{MC} \perp \overleftrightarrow{AC}$, $\because C \in \overline{MN}$

$\therefore \overline{NC} \perp \overleftrightarrow{AC}$,

In circle N, $\overline{AB}$ is a chord

, $\overline{NC} \perp \overleftrightarrow{AC}$

In circle N , $\because \overline{AB}$ is a chord

, $\therefore \overline{MN} \perp \overline{AB}$,

$\therefore$ C is a mid point of $\overline{AB}$

$\Delta\Delta$ AMC , BMC

$\because \overline{MC}$ is common side

, CA = CB

, m(∠ MCA) = m(∠MCB)

$\therefore \Delta$ AMC $\equiv \Delta$ BMC

$\therefore$ MA = MB

(5) $\because$ M , N are two intersecting circles, $\overline{AB}$ is the common chord.

$\therefore \overline{MN} \perp \overline{AB}$, $\therefore$ m (∠ AFN) = 90°

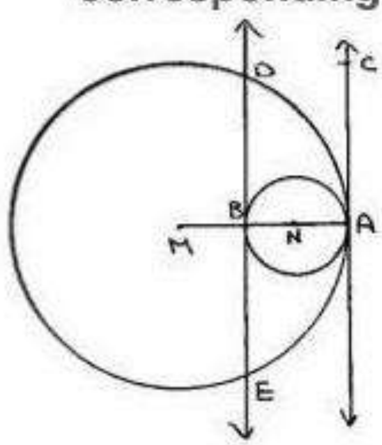
$\because$ E is a mid point of $\overline{CD}$

$\therefore \overline{FM} \perp \overline{CD}$, m (∠ CEF) = 90°

$\because$ m (∠ AFN) = m(∠ CEF) = 90° corresponding angles

$\therefore \overline{CD} \mathbin{//} \overline{AB}$

(6) $\because \overleftrightarrow{AC}$ is the common tangent



$\therefore \overleftrightarrow{MN} \perp \overleftrightarrow{AC}$

$\because$ B $\in$ MN , $\overleftrightarrow{ED}$ tangent (N)

$\therefore \overline{MN} \perp \overline{DE}$

$\therefore \overleftrightarrow{CA} \parallel \overleftrightarrow{DE}$

$\because \overline{MA} \perp \overline{DE}$ $\Rightarrow$ $\overline{DE}$ is a chord in circle M

$\therefore$ B is the mid point of $\overline{DE}$

$\therefore$ BD = BE

(7) construction $\therefore$ Draw $\overline{AX}$ and $\overline{DY}$

$\because \overrightarrow{AB}$ is a tangent to circle M at A

$\therefore \overline{MA} \perp \overrightarrow{AB}$, $\because \overline{AC} \parallel \overline{FD}$

$\therefore$ m($\angle$ AXE) = 90°

$\because \overrightarrow{DE}$ is a tangent to circle N at D

$\therefore \overline{ND} \perp \overrightarrow{DE}$, $\because \overline{AC} \parallel \overline{FD}$

$\therefore$ m ($\angle$ DYB) = 90°

$\therefore$ AXDY is a rectangle $\therefore$ AX = DY

$\because$ M and N are two congruent circles

$\therefore$ MA = ND $\therefore$ MX = NY

$\because \overline{MX} \perp \overline{EF}$, $\overline{NY} \perp \overline{BC}$

$\therefore$ EF = BC (1) (1st)

$\because$ Ay = XD (2)

$\because \overline{MX} \perp \overline{EF}$

$\therefore$ x is a mid point of $\overline{EF}$

similarly y is the mid point of $\overline{BC}$

$\because$ EF = BC $\therefore$ BY = XE (3)

Subtracting (3) from (2)

$\therefore$ AB = DE (2nd)

(8) construction: Draw $\overline{ML} \perp \overleftrightarrow{EF}$

$\because$ x and y are mid points of $\overline{AB}$ and $\overline{AC}$ respectively.

$\therefore \overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

$\because$ AB = CD

$\therefore$ MX = MY

Δ MXY is an isosceles Δ

$\therefore$ LX = LY (1)

$\because \overline{ML} \perp$ the chord $\overline{EF}$

$\therefore$ EL = LF (2)

By subtracting (1) from (2)

We get XE = YF

(9) $\because$ MN are two intersecting circles.

$\therefore \overline{MN} \perp \overline{AB}$

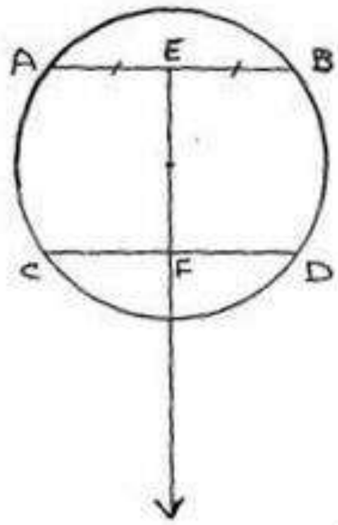
$\because$ x is the mid point of $\overline{AC}$

$\therefore \overline{NX} \perp \overline{AC}$

$\because$ AB = AC

$\therefore$ NX = NY

(10) $\because$ E is the mid point of $\overline{AB}$



$\therefore \overline{ME} \perp \overline{AB}$

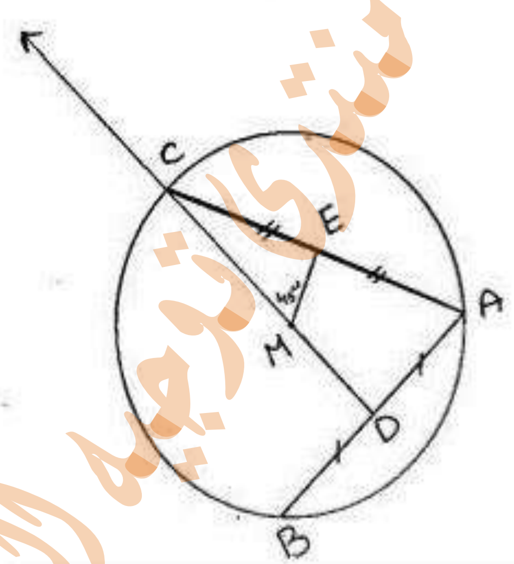
$\because \overline{AB} // \overline{CD}$

$\therefore \overline{MF} \perp \overline{CD}$

$\therefore$ F is a mid point of $\overline{CD}$

$\therefore$ FC = FD

(11) $\because$ E , D are the mid point of $\overline{AB}$ and $\overline{AC}$ respectively



$\therefore \overline{MD} \perp \overline{AB}$

$\overline{ME} \perp \overline{AC}$

In Δ EMF

$\because$ m ($\angle$ MEF) = 90° , EM = EF

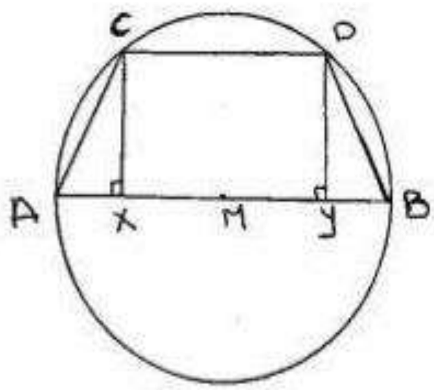
$\therefore$ m($\angle$ EMF) = $\frac{180-90}{2}$ = 45°

M ($\angle$ EMD) = 180° - 45° = 135°

In the quad ADME

m ($\angle$A) = 360° - (90° + 90° + 135°) = 45°

(12) constructions: Draw $\overline{AC}$ and $\overline{BD}$



$\because \overline{CD} \parallel \overline{AB}$

$\therefore$ m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{AC}$)

$\therefore$ AC = BD

$\Delta\Delta$ AXC and BYD

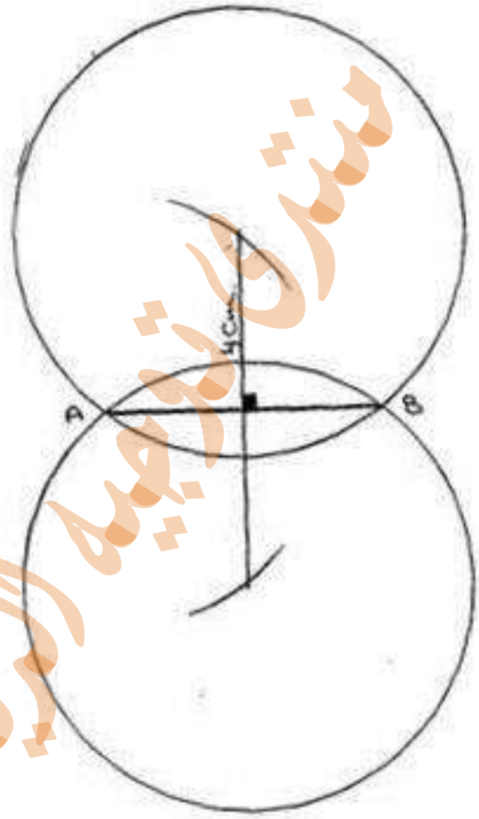
$\left\{\begin{array}{l} \text{M } (\angle DYB) = \text{m } (\angle CXA) = 90° \\ CX = DY \\ AC = DB \end{array}\right.$

$\therefore \Delta$ AXC $\equiv \Delta$ BYD

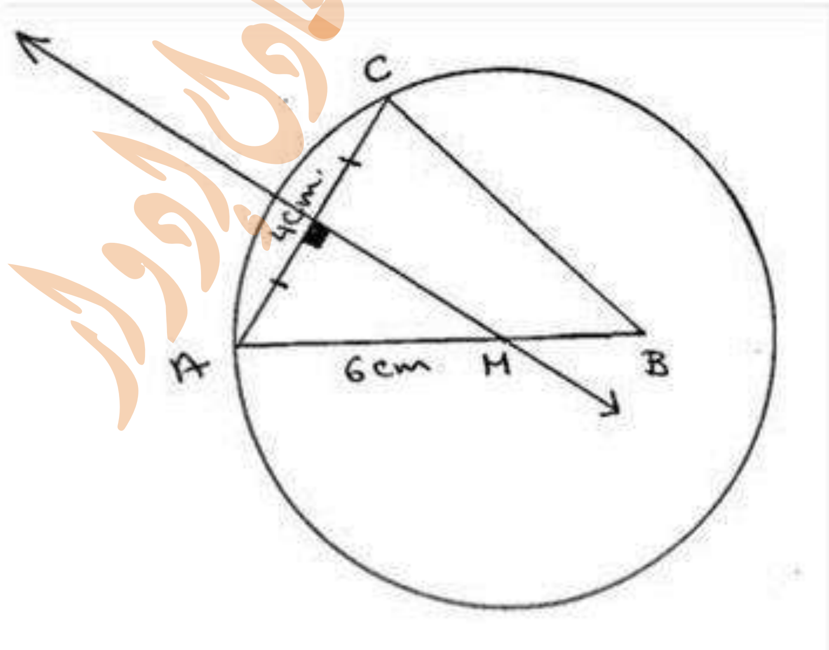
$\therefore$ AX = YB

(13) The distance is the length of the perpendicular from the centre of the circle to $\overline{AB}$

Distance = 4 cm.



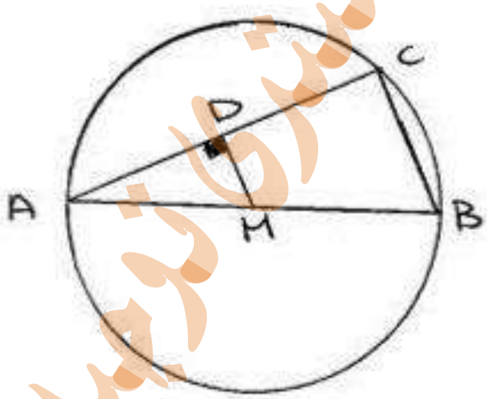
(14)



(15) $\because \overline{AB}$ is a diameter

$\therefore$ m ($\angle$ACB) = 90°

inscribed (semicirlie)



$\because$ m ($\angle$ ADM) = m ($\angle$ ACB) = 90°

(corresponding)

$\therefore \overline{DM} \,//\, \overline{CB}$

$\because \Delta$ ABC right angled at c

$\because$ m ($\angle$A)= 30°

$\therefore$ CB = $\frac{1}{2}$ AB

General Exercise on the 2nd unit

First:

(1) supplementary (2) equal in length

(3) central angle subtended by the same arc.

(4) (1) 132° (2) 264° (5) measure

(6) a) 36° b) 72° c) 144°

(7) acute angle (8) equal

(9) the inscribed angle subtended by this arc.

Second :Choose:

(1) c (2) c (3) d (4) d (5) d (6) d,a

Questions Part (2)

(1) Choose:

1- The angle of tangency included between ....

a- two chords   b- two tangents

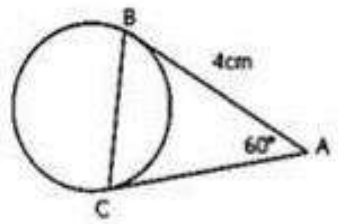
c- chord and tangent   d- chord and diameter

2- The number of tangents can be drawn from a point lies on a circle equals.

3- The number of common tangents can be drawn to two concentric circles equals ....

a-zero   b-one   c-two   d-three

4-in the opposite figure $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents , m ($\angle A$)= 60° If AB= 4 cm, then the length of $\overline{CB}$ equals ....



a-3cm   b-4cm   c-5cm   d-8cm

5- The number of common tangents can be drawn to two touching internally circles equals ....

a- one   b-two   c- three   d- four

6- Using the following figures choose the correct answer

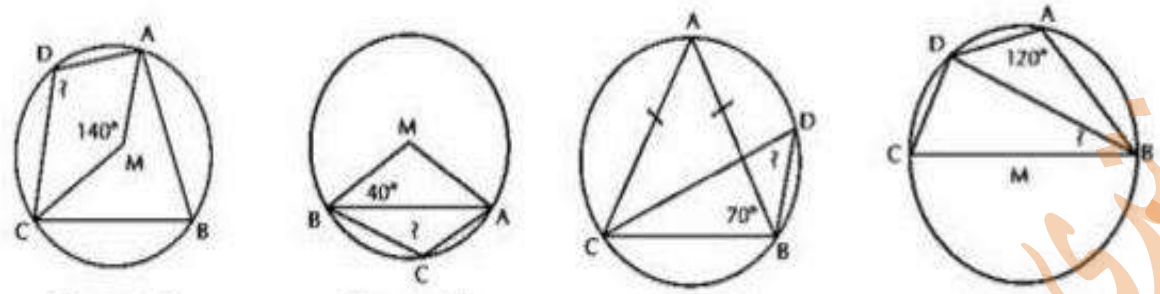


Figure (1) Figure (2) Figure (3) Figure (4)

In figure 1: A circle of center M , m(∠AMC) = 140° , then m(∠ADC) =.....°

a-40° b-70° c -110° d- 140°

In figure 2: if m(∠ABM)=40° , then m(∠ACB) = ...°

a-80° b- 100° c-130° d- 140°

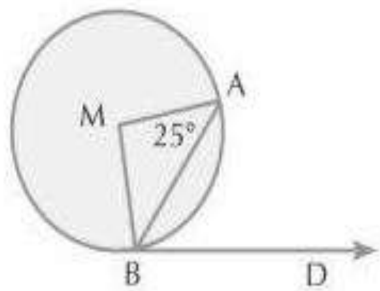
In figure 3: if m (∠ABC) = 70° , then m(∠BDC) =.........°

a- 20° b- 40° c- 60° d- 90°

In figure 4: if m(∠BAD) =120°, then m(∠CBD) =......°

a- 15° b- 30° c- 45° d- 60°

7- In the opposite figure:

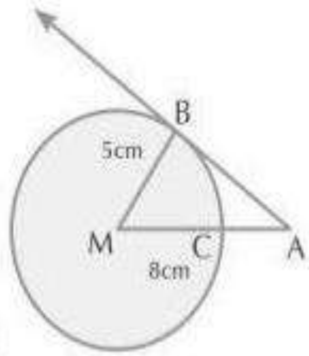


if $\overrightarrow{BD}$ is a tangent to the circle M,

m(∠BAM)=25° ,then m(∠ABD)= ....°

a-25° b-50° c-65° d-130°

8- In the opposite figure:



$\overrightarrow{BA}$ Is a tangent to the circle M ,

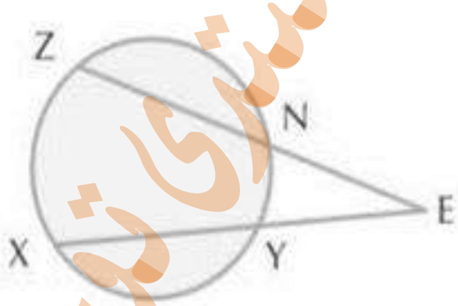
if MB = 5cm , AC= 8cm , then AB= .....cm

a- 5cm b- 10cm c-12cm d- 13cm

9- It is possible to draw a circle passing through the vertices of a ....

a- trapezium   b- rhombus   c- parallelogram   d- rectangle

10-in the opposite figure



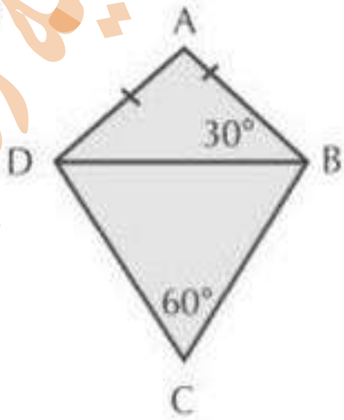
If m $(\overset{\frown}{XZ})$ =70°, m$(\overset{\frown}{YN})$=30°, then m(∠E)=.........°

a-20°   b-40°   c-50°   d-100°

(2) Answer the following questions

(1) (a) Prove that the two opposite angles in a cyclic quad are supplementary.

(b) In the opposite figure



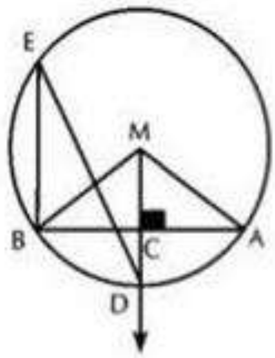
ABCD is a quadrilateral in which AB=AD,

m (∠ABD) = 30° and m(∠C) = 60° prove that

ABCD is a cyclic quad.

(2) ABCD is a cyclic quadrilateral in which $\overline{AB}$ // $\overline{CD}$, if E is the mia point of $\overset{\frown}{AB}$. prove that : EC= ED .

(3) In the opposite figure



$\overrightarrow{MC} \cap \overleftrightarrow{AB}$ = {c}, $\overline{MC}$ intersects the circle at D.

m(∠MAB) =20°, Find

1- m $(\overset{\frown}{AD})$ **2-** m(∠DEB).

(4) ABC is an acute angled triangle drawn inside a circle. draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and cuts the circle at E circle, then draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N . prove that:

1- ANDC is a cyclic quad.

2- m(∠BND) =m(∠BED)

(5) ABC is an equilateral triangle drawn inside a circle, D is a point on the arc AB , E is a point on $\overline{DC}$ such that AD = DE . Prove that:

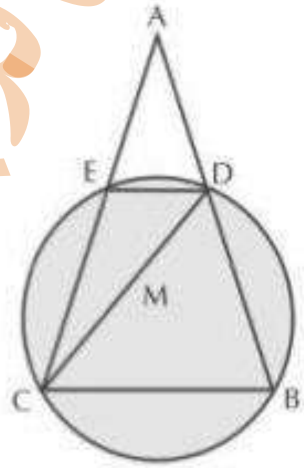
1- ADE is an equilateral triangle.

2- $\overline{DB} // \overline{AE}$

3- m(∠DCB) = m(∠EAC)

4- DB=EC

(6) In the opposite figure :

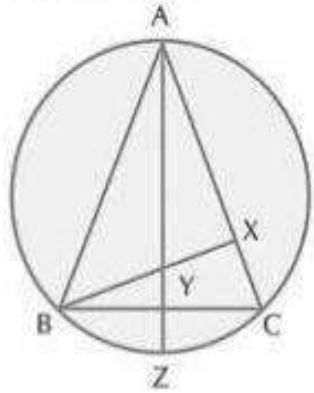


ABC is a triangle in which AB =AC. $\overline{BC}$ is a chord in the circle M, if $\overline{AB}$, $\overline{AC}$ cut the circle at D, E prove that: $\overline{BC} // \overline{DE}$ and if m(∠DCA) = 30° and m(∠A) =50°, find:

1-m(∠BEC) 2- m(∠BMC) 3- m(∠CDE)

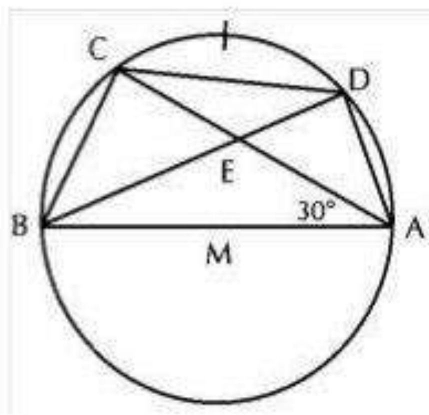
(7) (a) prove that the angles subtended by the same arcs in the circle are equal in measure .

(b) In the opposite figure ABC is a triangle in a circle, $\overline{BX} \perp \overline{AC}$,$\overrightarrow{AY} \perp \overline{BC}$ Cuts it at Y and cuts the circle at Z, prove that:



1- ABYX is a cyclic quad. 2- $\overline{BC}$ bisects ∠XBZ

(8) In the opposite figure

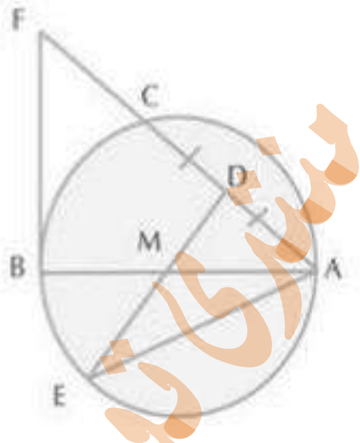


$\overline{AB}$ is a diameter of a circle M, C∈ the circle, m(∠CÂB)= 30°,D is the mid-point of the arc $\overset{\frown}{AC}$ and $\overline{DB} \cap \overline{AC}$={E}.

1- find m(∠BDC), m(∠ABD)

2-prove that Δ ABE is an isosceles triangle.

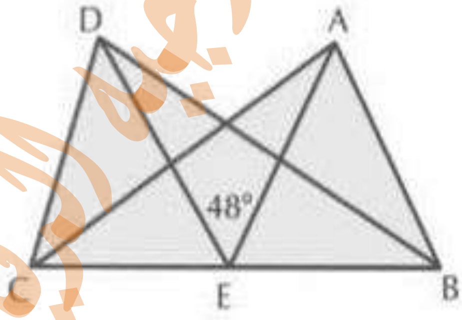
(9) In the opposite figure



$\overline{AB}$ is a diameter of a circle M,D is the mid-point of the arc $\overline{Ac}$ Draw $\overrightarrow{DM}$ to cut the circle at E, $\overrightarrow{BF}$ is a tangent to the circle to cut $\overrightarrow{AC}$ at F. prove that :

1-MBFD is a cyclic quad. **2**-$\overline{DE}$ //$\overline{BC}$.

(10) In the opposite figure



m (∠BAC) =m (∠BDC) =90°

E is the mid-point of $\overline{BC}$ and m (∠AED) =48°

1-find m (∠ABD)

2-prove that: (a) m (∠ABD) = m (∠ACD)

(b) m (∠AEC) = 2m(∠ABC)

(11) ABCD is a quadrilateral drawn in a circle, draw $\overrightarrow{EF}//\overrightarrow{CB}$ to cut $\overline{CD}$at E cuts $\overline{AB}$ at F, $\overrightarrow{DF}$ ∩ $\overrightarrow{CB}$ ={x} .prove that:

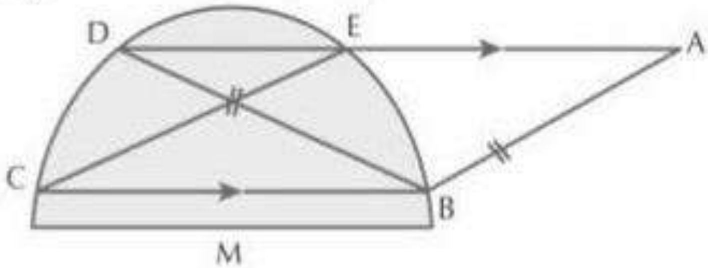
1- AFED is a cyclic quad.

2- m(∠BXF) = m (∠EAD)

(12) A is a point outside a circle, draw $\overrightarrow{AB}$to cut the circle at B,C respectively, then draw $\overrightarrow{AD}$ to cut the circle at D,E respectively, if AC= AE prove that:

1-$\overline{BD}//\overline{CE}$ 2- m($\widehat{BC}$) =m($\widehat{ED}$)

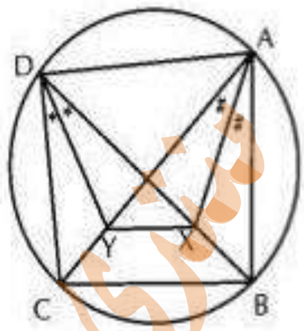
(13) In the opposite figure



A semicircle of center M, $\overline{AD}$ //$\overline{BC}$,

Prove that ABCE is a parallelogram.

(14) In the opposite figure

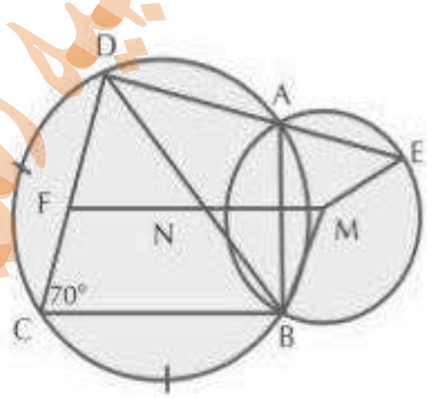


ABCD is a quadrilateral in a circle M, $\overline{AX}$ bisects ∠BAC, $\overline{DY}$ bisects ∠BDC, prove that:

1- AXYD is a cyclic quad.

2- $\overline{XY} // \overline{BC}$

(15) In the opposite figure



m(∠C)=70° , the length of $\overset{\frown}{CD}$ = the length of $\overset{\frown}{BC}$

$\overrightarrow{MN} \cap \overline{CD}$= {F} and $\overrightarrow{DA}$ ∩ the circle = {E}. find

M(∠BDC) , m(∠BAD) and m(∠BME)

(16) $\overline{AB}$ is a diameter of a circle M, D∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$.draw the tangent $\overrightarrow{DC}$at C, draw $\overrightarrow{CB}$,if E ∈ $\overrightarrow{CB}$such that DE=DC prove that:

1-ACDE is a cyclic quad .

2- $\overline{AE}$ is the diameter of the circumcircle of the figure ACDE.

3- $\overrightarrow{DE}$ is the tangent of the circumcircle of the triangle ABE.

Model Answers Part (2)

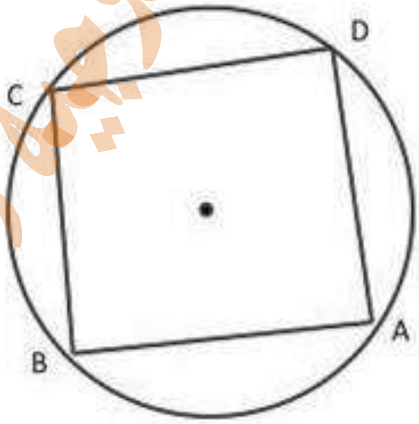
(1) Choose:

1) c 2) one 3) a 4) b 5) a

6) c , c , b , b 7)c 8) c 9)d 10) a

Answer the Following P.170

(1) (a)Given:



ABCD is cyclic quad .

R.T.P. $m(\angle A)+m(\angle C)= 180^\circ$

$m(\angle B)+ m(\angle D)=180^\circ$

proof : $\because m(\angle A)=\frac{1}{2}m(\widehat{BCD})$ and $m(\angle C)=\frac{1}{2}m(\widehat{BAD})$

$\because m(\angle A) + m(\angle C)=\frac{1}{2}[m(\widehat{BCD})+ m(\widehat{BAD})] =\frac{1}{2}$ m. of the circle

$= \frac{1}{2}\times 360^\circ =180^\circ$ similarly $m(\angle B) +m(\angle D) =180^\circ$

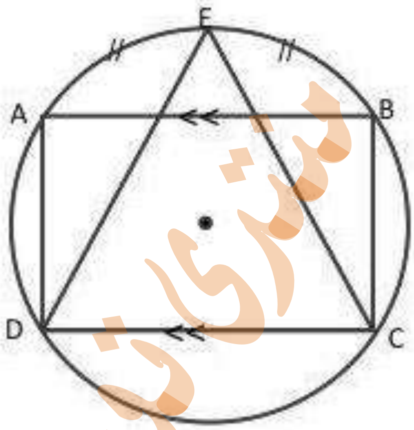
(b) $\because \Delta$ ABD is an isosceles Δ

$\because m(\angle ABD) =30^\circ$

$\therefore m(\angle A) =180^\circ - (30^\circ + 30^\circ)=120^\circ$

$\because m(\angle A) + m(\angle C) =120^\circ+60^\circ =180^\circ$

$\therefore$ ABCD is acyclic quad.



(2) $\overline{AB} // \overline{CD}$ $\therefore$ m($\overset{\frown}{BC}$) = m ($\overset{\frown}{AD}$) $\boxed{1}$

$\because$ E is the midpoint of (AB)

$\therefore$ m ($\overset{\frown}{BE}$) = m ($\overset{\frown}{AE}$) $\boxed{2}$

by adding $\boxed{1}$ and $\boxed{2}$

$\therefore$ m ($\overset{\frown}{CE}$) = m ($\overset{\frown}{DE}$)

$\therefore$ EC =ED

(3) $\because$ $\overline{MC} \perp \overline{AB}$, $\because$ m($\angle$A) =20°

$\therefore$ m($\angle$AMC) = 180 – (90+20) =70°

($\angle AM$D) is a central angle subtended by are ($\overset{\frown}{AD}$)

$\therefore$ m ($\overset{\frown}{AD}$) =70° , $\because$ $\overline{MC} \perp \overline{AB}$ $\therefore$CA =CB

$\Delta\Delta$ MCA and MCB

$\because MA = MB = r$

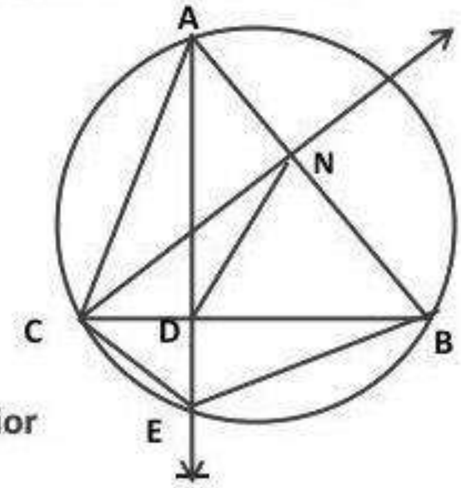
$\because AC = BC$

$\because$ $m\,(\angle MCA) = m\,(\angle MCB) = 90°$

$\therefore$ ΔMCA $\equiv$ Δ MCB

$\therefore$ $m\,(\angle AMC) = m\,(\angle BMC)$ =70° $\therefore m\,(\angle BED) = \frac{1}{2} m\,(\angle BMD)$

=70÷2 =35° (inscribed and central angles subtended by the same are ($\overset{\frown}{BD}$))



(4) $\because \overrightarrow{AD} \perp \overline{BC}$, $\because \overrightarrow{CN} \perp \overline{AB}$

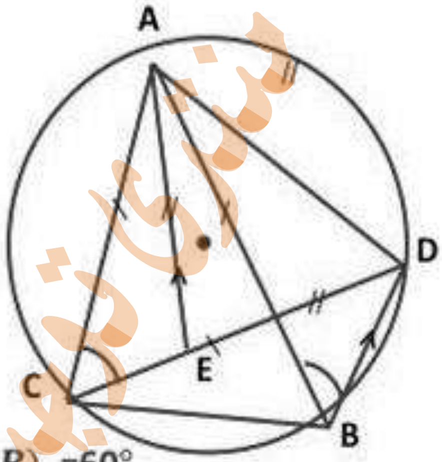
$\therefore$ $m\,(\angle ANC) = m\,(\angle ADC) = 90°$

Subtended dry the chord $\overline{AC}$

and on one side of it $\therefore$ANDC is cyclic quad.

$\therefore m\,(\angle BND) = m\,(\angle ACD)$ exterior = opposite in terior

$\because$ $(\angle BEA)$ and $(\angle BCA)$ two inscribed angles subtended by same are ($\overset{\frown}{AB}$)



$\therefore m\,(\angle BED) = m\,(\angle BND)$

(5) $\because$ Δ ABC is an equilateral Δ

$\therefore m\,(\angle B)$= 60°

$\because \angle B, \angle D$ are two inscribed angles

subtended by the same are ($\overset{\frown}{AC}$) $\therefore m\,(\angle D) = m\,(\angle B)$ =60°

$\because$ AD =DE $\therefore m\,(\angle DAE) = m\,(\angle AED)$ $=\frac{180-60}{2}$ =60°

$\therefore$ Δ ADE is an equilateral Δ $\boxed{1}$

$\angle CDB$ and $\angle CAB$ are two inscribed angles subtended by the same are $\overset{\frown}{CB}$

$\therefore m\,(\angle CDB) = m\,(\angle CAB)$ =60° $\because m\,(\angle AED)$ =60° (equilateral Δ)

$\because m\,(\angle AED) = m\,(\angle EDB)$ =60° alternate angles

$\therefore \overline{AE} // \overline{DB}$ $\boxed{2}$

$\because$ Δ ADE is an equilateral Δ $\therefore m\,(\angle AED)$ =60° (exterior angle)

$\therefore$ m($\angle$AED) =m($\angle EAC) + m\,(\angle ECA)$ $\Rightarrow \angle EAC + \angle ECA = 60° \rightarrow 1$

$\because \angle DCB + \angle ECA = 60° \rightarrow 2$

From 1, 2 $\therefore m(\angle DCB) = m\,(\angle EAC) \rightarrow 3$

In $\Delta\Delta$ ADB ,AEC $\because \angle BCD$, $\angle BAD$ *are* 2 inscribed angles sub tended by same are ($\overset{\frown}{BD}$)

$\therefore m(\angle BCD) = m\,(\angle BAD) \rightarrow 4$

From 3,4

$\therefore m(\angle BAD) = m\,(\angle EAC)$ & AB =AC & AE = AD

$\therefore \Delta\, ADB \equiv \Delta\, AEC$

$\therefore$ DB = EC

(6) In $\Delta\, ABC$, $\because AB = AC$ $\therefore$ Δ ABC is an isoscelesΔ.

$\therefore m(\angle ABC) = m\,(\angle ACB)$

$\therefore m(\widehat{DEC}) = m\left(\widehat{EDB}\right) \rightarrow 1$

Subtract m ($\widehat{ED}$) from 1 $\therefore m(\widehat{EC}) = m(\widehat{DB})$ $\therefore \overline{BC} \,//\, \overline{DE}$

$\because m(\angle A) = 50°$, AB =AC

$\therefore m(\angle ACB) = \frac{180° - 50°}{2} = 65°$ $\because$ m $(\angle\, DCA) = 30°$

$\therefore m(\angle DCB) = 35°$ $\because m(\angle\, DBC) = 65°$

$\therefore In\,\Delta\, DBC$ $\therefore m(\angle BDC) = 180° - (35° + 65°) = 80°$

$\because \angle\, BDC, \angle\, BEC$ subtended by same are $\widehat{BC}$

$\therefore m(\angle BEC) = 80°$, $\because m\,(\angle\, BHC), m\,(\angle\, BDC\,)$ are central, inscribed angles subtended by same are $\widehat{BC}$.

$\therefore m(\angle BMC) = 2m\,(\angle BDC) = 2 \times 80°$

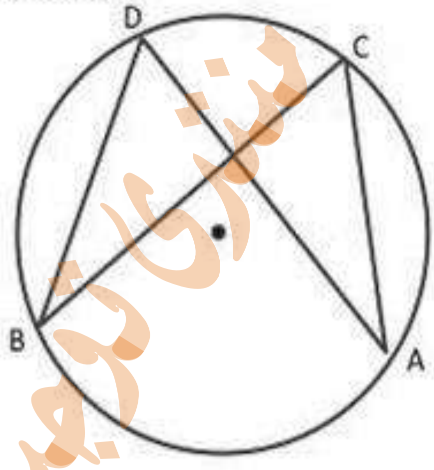
$\therefore m(\angle BMC) = 160°$

$\because \overline{AB}$ is a straight line , $\because$ m $(\angle\, BDC) = 80°$

$\therefore m(\angle ADC) = 100°$, $\because \overline{ED} \,//\overline{BC}$

$\therefore m(\angle ADE) = 65°$

$\therefore m(\angle CDE) = 100° - 65° = 35°$



Given: $\angle C, \angle D$ are inscribed angles subtended by same are $\overset{\frown}{AB}$

R.t.P.: m ($\angle c$) =m($\angle D$)

$\because m(\angle C) = \frac{1}{2} m\, \widehat{AB}$, $m(\angle D) = \frac{1}{2} m\, \widehat{AB}$

$\therefore$ m ($\angle c$) =m($\angle D$)

(¡¡) In Δ ABC $\because \overline{BX} \perp \overline{AC}$ $\therefore$ m(∠AXB) =90°

$\because \overline{AY} \perp \overline{BC}$ $\therefore$ m(∠AYB) =90°

$\because m(\angle AXB) = m(\angle AYB)$

& drawn on one side of its sides as a base , on one side

$\therefore$ AXYB is a cyclic quadrilateral

$\therefore m(\angle XAY) = m(\angle YBX)$ &

$\because (\angle XAY), (\angle XBZ)$ are subtended by same are $\widehat{ZC}$

$\therefore m(\angle XBY) = m(\angle YBZ)$

$\therefore \overline{BC}$ bisects ∠XBZ

(8) $\because \overline{AB}$ is a diameter $\therefore$ m(∠ACB) =90°

$\because$m(∠BAC) =30° & m(∠BDA) =90°

$\therefore$ m(∠ABC) =180°-(90°+30°)=60°

$\because$ D is midpoint of are $\overset{\frown}{AC}$ $\therefore \widehat{AD} = \widehat{DC}$

$\therefore$ m(∠ABD) = m(∠DBC) = $\frac{60°}{2}$ =30°

$\therefore$ m(∠ABD) =30°

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m(∠ABC)+m (∠ADC) =180°

$\therefore$ 60° +m(∠BDC)+ m(∠BDA) =180°

$\therefore$ m(∠BDC) =180°- (60° +90°) =30°

$\therefore$ m(∠BDC) =30°

In ΔΔ BCE ,ADE

$\because$ m(∠BCA) = m(∠BDA) =90°

& m(∠CBD) = m(∠CAD) (subtended by same Arc $\overset{\frown}{CD}$)

& m(∠BEC) = m(∠AED) (V. O .A)

$\therefore \Delta BCE \equiv \Delta ADE$ $\therefore$ BE = EA

(9) $\because$ $\overrightarrow{BF}$ is a tangent to circle M (with a diameter)

$\therefore$ m (∠ABF) =90° $\boxed{1}$

& $\because$ D is midpoint of $\overline{AC}$ $\therefore \overline{MD} \perp \overline{AC}$

$\therefore$ m(∠MDF) =90° $\therefore$ m(∠ABF) + m(∠MDF) =180°

$\therefore$ MBFD is a cyclic quadrilateral

$\because$ $\overline{AB}$ is a diameter .

$\therefore$ M is midpoint of $\overline{AB}$ & D is midpoint of $\overline{AC}$

$\therefore \overline{DE} // \overline{BC}$

(10) construction : Draw $\overline{AD}$

Proof: $\because$ m(∠BAC) = m(∠BDC) =90°

$\therefore$ figure ABCD is a cyclic quadrilateral and $\overline{BC}$ is a diameter in the circumclrcle of it

$\because$ E is midpoint of $\overline{BC}$

$\therefore$ E is Centre of circle which passes through points A, B, C and D.

$\therefore$ m(∠ABD) $=\frac{1}{2}$ m(∠AED) =24°

inscribed angle & central angle of same arc ($\overset{\frown}{AD}$)

∵ ABCD is a cyclic quadrilateral

∴ m(∠ABD) = m(∠ACD) (drawn on $\overline{AD}$& on one side of it)

& ∵ m(∠BAD) = m(∠BDC) =90°

∴ $\overline{BC}$ is a diameter & ∵ E is the midpoint of $\overline{BC}$

∴ E is the center of circum circle of ABCD

∴ ∠ ABC & ∠ AEC are inscribed & central angle subtended by arc $\overset{\frown}{AC}$

∴ m(∠AEC) = 2m(∠ABC)

(11) ∵ $\overline{EF}//\overline{BC}$

∴ m(∠FED) = m(∠BCD) → $\boxed{1}$ (corresponding angles)

& ∵ ABCD is acyclic quad .

∴ m(∠FAD)+ m(∠BCD) =180° → $\boxed{2}$

From 1 & 2

∴ m(∠FAD) + m(∠FED) =180°

∴ AFED is a cyclic quad .

∵ $\overline{EF}//\overline{BC}$

∴ m(∠EFD) = m(∠BXF) (corresponding angles)

& m(∠EFD) = m(∠EAD) (drawn on $\overline{ED}$ and on side of it) &

(AFED is a cyclic quad)

∴ m(∠BXF) = m(∠EAD)

(12) In Δ ABC ∵ AE =AC

∴ Δ ABC is an isosceles Δ

∴ m(∠ACE) = m(∠AEC) → $\boxed{1}$

∵ CBDE is acyclic quad .

$\therefore m(\angle CED) + m(\angle CBD) = 180°$
& $m(\angle ECB) + m(\angle BDE) = 180°$ } (2)

From 1& 2

$\therefore m(\angle DBC) + m(\angle BCE) = 180°$ & $m(\angle BDE) + m(\angle DEC) = 180°$

$\therefore \overline{BD} // \overline{CE}$ $\therefore m(\widehat{BC}) = m(\widehat{ED})$

(13) In Δ ABD $\because$ AB = BD

$\therefore m(\angle A) = m(\angle BDA)$

$\because m(\angle BCE) = m(\angle BDE)$ (2 inscribed angles subtended by $\widehat{BE}$)

$\therefore m(\angle A) = m(\angle BCE) \rightarrow \boxed{1}$

$\because \overline{AD} // \overline{BC} \therefore \angle$ A supplements $\angle ABC$, $\angle$ BCE supplements $\angle$ AEC

$\therefore m(\angle ABC) = m(\angle AEC) \rightarrow \boxed{2}$

From 1&2

$\therefore$ ABCE is a parallelogram.

(14) $\because$ ABCD is a cyclic quad.

$\therefore m(\angle BAC) = m(\angle BDC)$ (drawn on $\overline{BC}$ & on one side of it)

$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle BDC)$

$\therefore m(\angle XAY) = m(\angle XDY)$ but they are draw on $\overline{XY}$ & on one side of it

$\therefore$ AXYD is a cyclic quad $\because$ ABCD is a cyclic quad

$\therefore m(\angle CBD) = m(\angle CAD) \rightarrow \boxed{1}$ (drawn on $\overline{CD}$ & on one side of it)

$\because$ AXYD is a cyclic quad

$\therefore m(\angle YXD) = m(\angle YAD) \rightarrow \boxed{2}$ from 1&2

$\therefore m(\angle CBD) = m(\angle YXD)$ similarly $\therefore m(\angle BCA) = m(\angle XYA)$

$\therefore \overline{XY} // \overline{BC}$

(15) $\because \widehat{CD} = \widehat{CB}$ $\quad \therefore CD = CB$

$\therefore \Delta$ CBD is an isosceles Δ

$\therefore m(\angle BDC) = \frac{180° - 70°}{2}$

$\therefore m\,(BDC) = 55°$

$\because$ ABCD is a cyclic quad

$\therefore m(\angle BCD) + m(\angle BAD) = 180°$

$\therefore 70° + m(\angle BAD) = 180°$

$\therefore m(\angle BAD) = 110°$

$\because \overline{DE}$ is a straight line

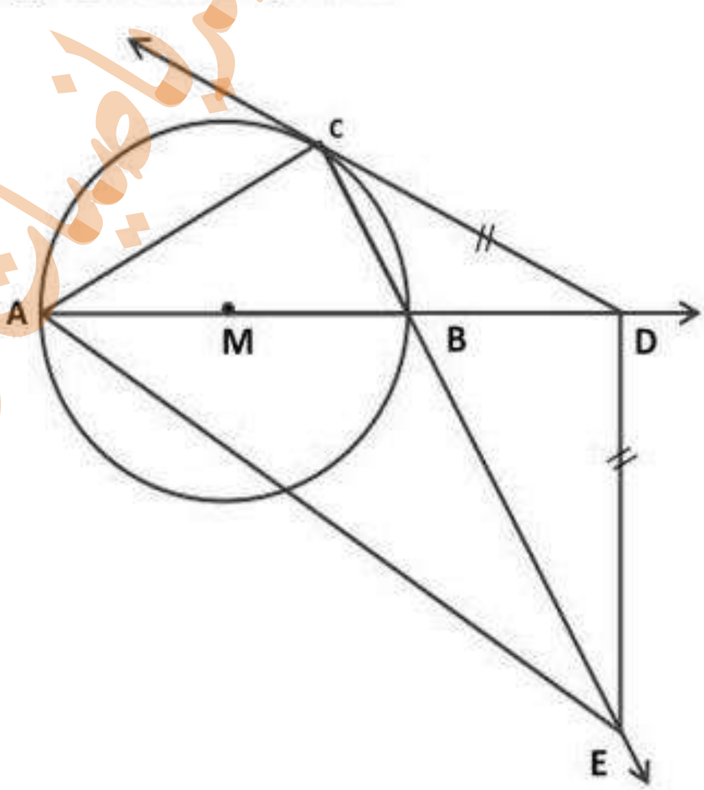
$\therefore m(\angle EAB) = 70°$

$\angle$ EAB & $\angle$ EMB are central & inscribed angle subtended by arc $\widehat{BE}$

$\therefore m(\angle BME) = 70° \times 2 = 140°$

$(m(\angle BME) = 2\, m(\angle BAE))$

(16) $\because$ CD = DE



$\therefore \Delta CDE$ is an isosceles Δ

$\therefore m(\angle DCE) = m(\angle DEC)$

$\because \overrightarrow{DC}$ is a tangent at C

$\therefore m(\angle DCB) = m(\angle CAD)$

$\therefore m(\angle CAD) = m(\angle CED)$

(drawn on $\overline{CD}$ & on one side of it)

$\therefore$ ACDE is a cyclic quad.

$\because \overline{AB}$ is a diameter in circle M

$\therefore m(\angle ACE) = 90°$

$\therefore \overline{AE}$ is a diameter in circumcircle of figure ACDE

$\because$ ACDE is a cyclic quad

$\therefore m(\angle DCE) = m(\angle DAE)$

(drawn on $\overline{DE}$ and on one side of it)& $\because m(\angle DCE) = m(\angle DEC)$

$\therefore m(\angle DEC) = m(\angle DAC)$

$\therefore$ DE is a tangent to circum circle of Δ ABE.

REVISION 1

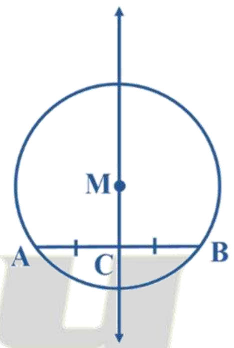
Corollary (1)

The straight line passing through the center of a circle and the midpoint of any chord of it is perpendicular to this chord.

if $\overline{AB}$ is a chord of a circle M and $\overleftrightarrow{MC}$ is drawn

$\because$ C is the midpoint of $\overline{AB}$

$\therefore$ $\overleftrightarrow{MC} \perp \overline{AB}$



Corollary (2)

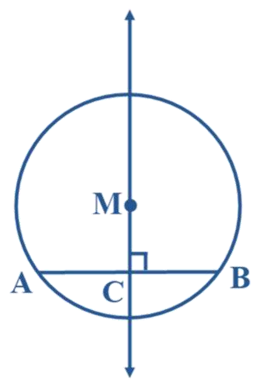
The straight line passing through the center of a circle and perpendicular to any chord of it bisects this chord.

$\overline{AB}$ is a chord of a circle M and $\overleftrightarrow{MC}$ is drawn

$\because$ $\overleftrightarrow{MC} \perp \overline{AB}$

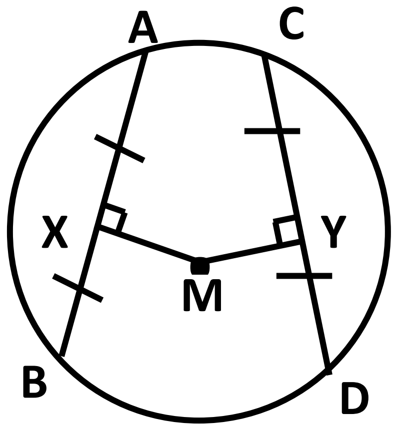
$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore$ $\overline{AC} = \overline{CB}$



Theorem

If chords of a circle are equal in length , then they are <u>equidistant</u> from the centre.



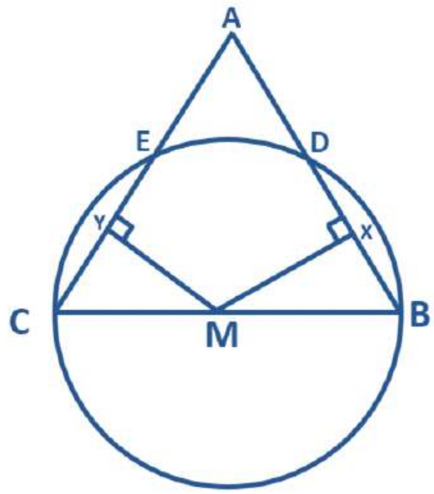
$\because \overline{AB} = \overline{CD}$

$\therefore \overline{MY} = \overline{MX}$

Important example

ABC is a triangle in which AB = AC. circle M was drawn with diameter $\overline{BC}$ intersecting $\overline{AB}$ at D and $\overline{AC}$ at E ,

$\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$ **prove that : BD = CE**



Solution

In $\Delta CMY, \Delta BMX$

$\because \overline{MB} = \overline{MC}$ (two radii)

$\because m(\angle CYM) = m(\angle BXM) = 90^\circ$

$\because m(\angle B) = m(\angle C)$ (because AB=AC)

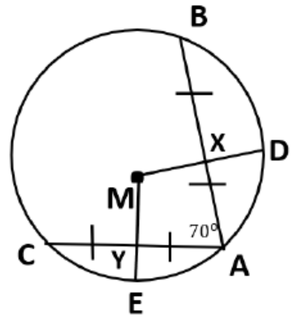
$\therefore \Delta CMY \equiv \Delta BMX$

$\therefore MX = MY$, but $\overline{MX} \perp \overline{BD}$ and $\overline{MY} \perp \overline{CE}$

$\therefore \overline{BD} = \overline{CE}$

EXAMS QUESTIONS

1) In the opposite figure:



AB and AC are two equal chords in circle M , X and Y are the midpoint of AB and AC m(∠A) = 70°

a) Find m(∠DME)

b) Prove that XD = YE

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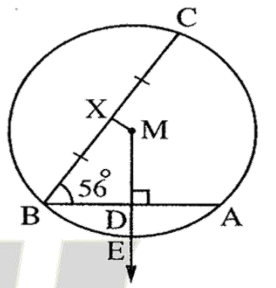
2) In the opposite figure:



BC and AB are two chords in circle M , X and D are the midpoint of AB and AB m(∠B) = 56° , MD = 8cm

a) Find m(∠XMD)

b) Find the length of DE

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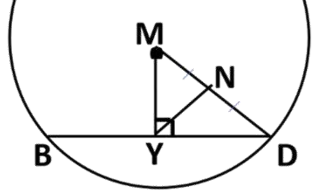
3) In the opposite figure:



YN=3cm , $\overline{MY} \perp \overline{BD}$, N is a midpoint of MB

Find area of circle M ($\pi = \frac{22}{7}$)

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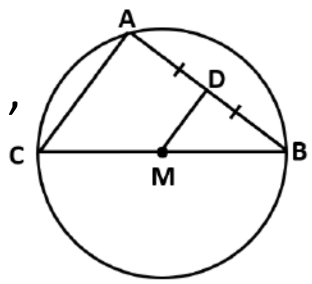
4) In the opposite figure:

$\overline{AB}$ is a chord in a circle , M , $\overline{BC}$ is a diameter on it ,
D is the midpoint of $\overline{AB}$

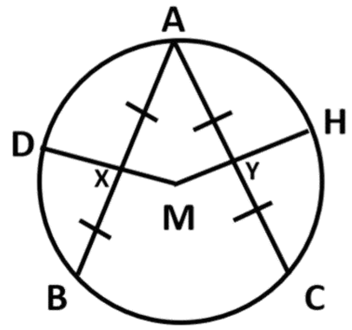
***1)* Prove that $\overline{MD}$ // $\overline{AC}$**

***2)* Find m $(\angle A)$**



5) In the opposite figure:

AB = AC , X is the mid-point of $\overline{AB}$, Y is the mid-point of $\overline{AC}$ **prove that: DX = HY**

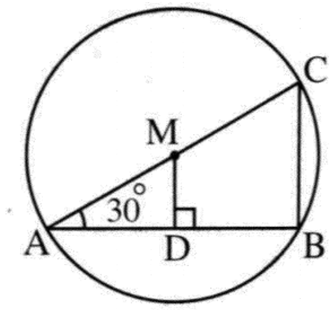


6) In the opposite figure:

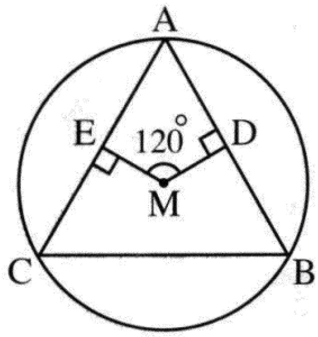
A circle M , $\overline{MD} \perp \overline{AB}$, m($\angle$A) = 30°

***1)* Prove that $\overline{MD}$ // $\overline{AC}$**

***2)* Find m $(\angle A)$**



7) In the opposite figure:



A circle M , $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$, where MD = ME

m(∠EMD) = 120°

prove that Δ ABC is equilateral.

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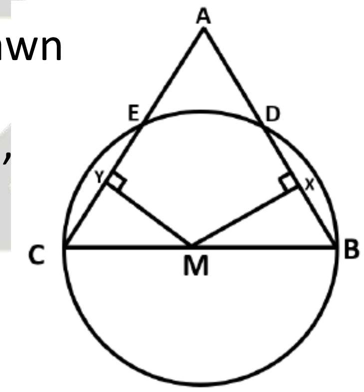
8) In the opposite figure:



ABC is a triangle in which AB = AC. circle M was drawn with diameter $\overline{BC}$ intersecting $\overline{AB}$ at D and $\overline{AC}$ at E , $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$ **prove that : BD = CE**

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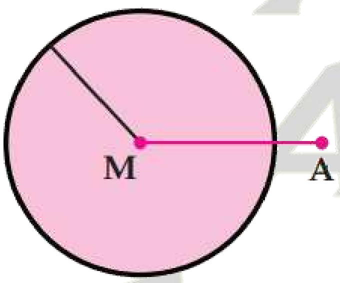
REVISION 2

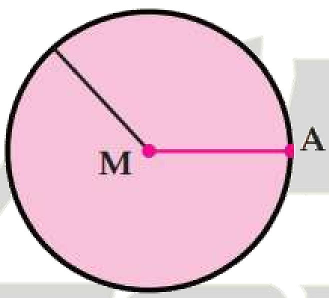
First : Position of a point with respect to a circle.

1 A is outside the circle



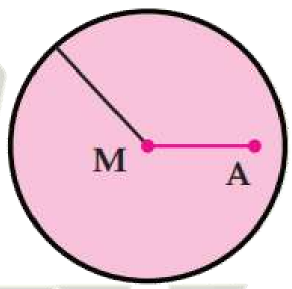
So : MA > r
and vise versa

2 A is on the circle



So : MA = r
and vise versa

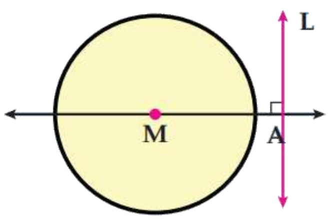
3 A is inside the circle



So : MA < r
and vise versa

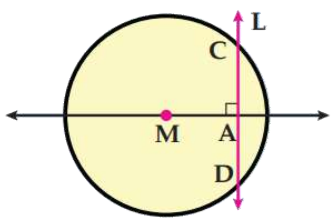
Second: Position of a straight line with respect to a circle :

1 the straight line L is located outside the circle M
L ∩ circle M = ϕ



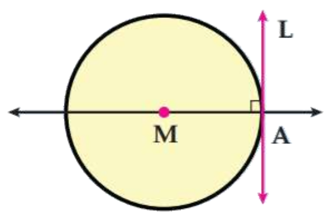
So : MA > r
and vise verse

2 the straight line L is a secant to the circle M
L ∩ circle M = {C, D}



So : MA < r
and vise verse

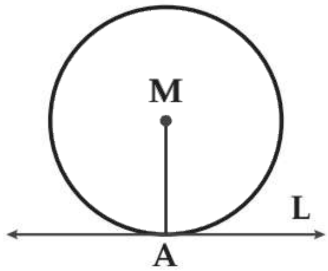
3 the straight line L is tangent to circle M
L ∩ the circle = {A}



So : MA = r
and vise verse

Fact

A tangent to a circle is perpendicular to the radius at its point of tangency.



$\because \overleftrightarrow{AL}$ is a tangent

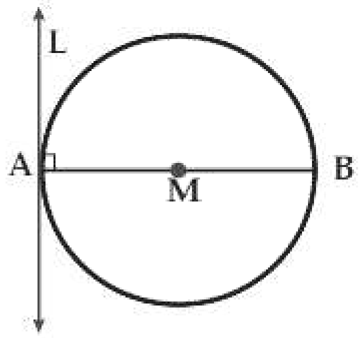
$\because \overline{AM}$ is a radius

$\therefore \overleftrightarrow{AL} \perp \overline{\mathrm{AM}}$

$\therefore$ m (∠ MAL) = 90°

Fact

If a straight line is perpendicular to a diameter of a circle at one of its endpoints, then it is a tangent to the circle.



$\because \overleftrightarrow{AL} \perp \overline{\mathrm{AM}}$

$\because \overline{AM}$ is a radius

$\therefore \overleftrightarrow{AL}$ is a tangent

1) Choose the correct answer :

1) If M circle with radius length = 4 cm and A is a point in its plane, MA = 3 cm, then A is .... circle M.

(inside – on – outside)

2) If M circle with radius length = 4 cm and A is a point in its plane, MA = 4 cm, then A is .... circle M.

(inside – on – outside)

3) If M circle with radius length = 4 cm and A is a point in its plane, MA = 5 cm, then A is .... circle M.

(inside – on – outside)

4) A tangent to a circle is ....the radius at its point of tangency.

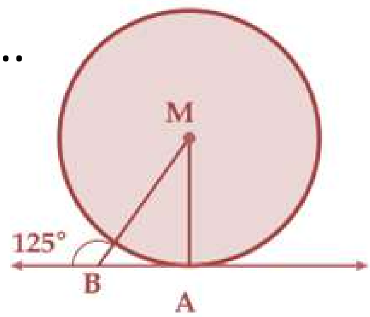
(perpendicular to – parallel to – bisects)

5) If a straight line is perpendicular to a diameter of a circle at one of its endpoints, then it is a .... to the circle.

(axis of symmetry – tangent – chord)

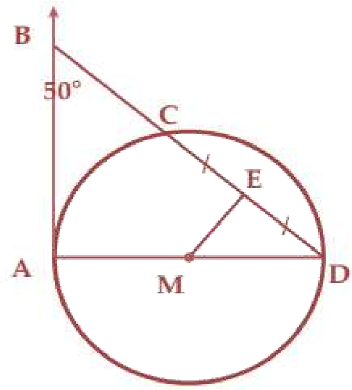
6) In the opposite figure: $m\,(\angle AMB) =$ ....

(25° – 35° – 45°)



2) In the opposite figure:

AB is a tangent to the circle M, E is the midpoint of the chord CD , m $(\angle ABC) = 50^\circ$



Find : $m\,(\angle AME)$

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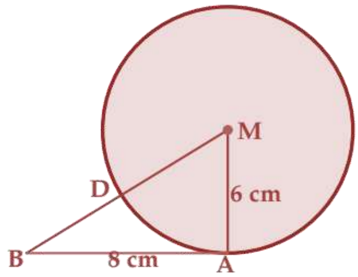
3) In the opposite figure:

AB is a tangent to the circle M, AM = 6 cm
AB = 8 cm



Find : **The length of DB**

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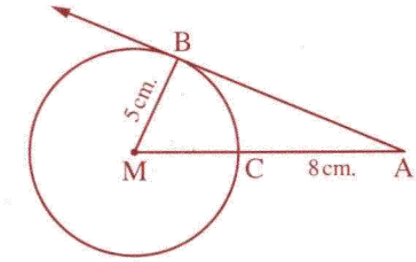
4) In the opposite figure:

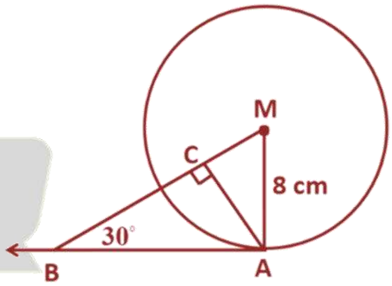


AB is a tangent to the circle M, AC = 8 cm

MB = 5 cm

Find : The length of MC

5) In the opposite figure:

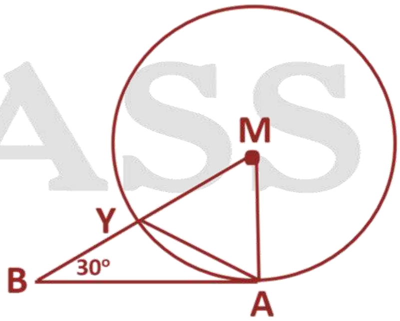


AB is a tangent to the circle M at A and

AM = 8 cm , m ($\angle$ ABM) = 30°

Find **the length of each : $\overline{AB}$ and $\overline{AC}$**

6) In the opposite figure:



AB is a tangent to the circle M at A and

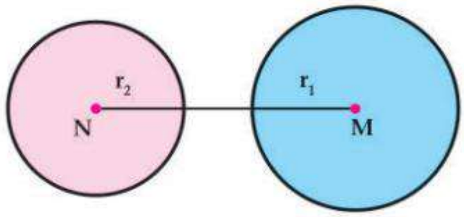
m ($\angle$ ABM) = 30°

prove that : $\overline{AY}$ = $\overline{BY}$

REVISION 3

Position of a circle with respect to another circle

1) $MN > r_1 + r_2$ the two circles are **distant**



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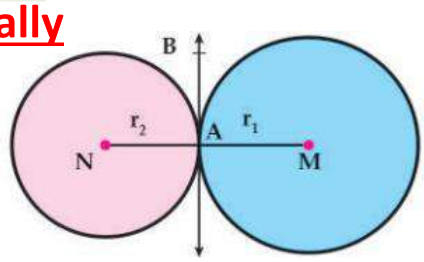
2) $MN = r_1 + r_2$ the two circles are **touching externally**



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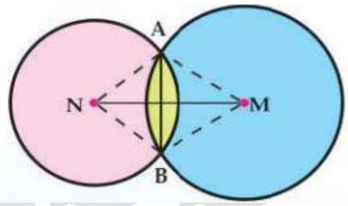
3) $r_1 - r_2 < MN < r_1 + r_2$ the two circles are **intersecting**



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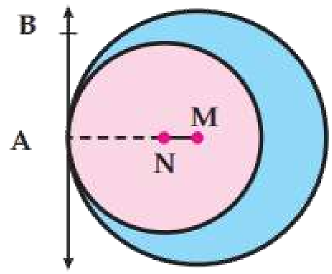
4) $MN = r_1 - r_2$ the two circles are **touching internally**



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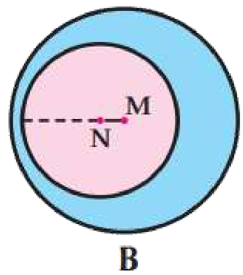
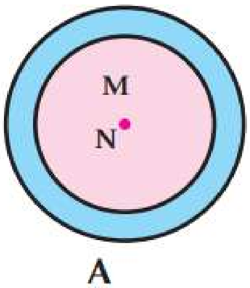
5) $MN < r_1 - r_2$ the two circles are **one inside the other**



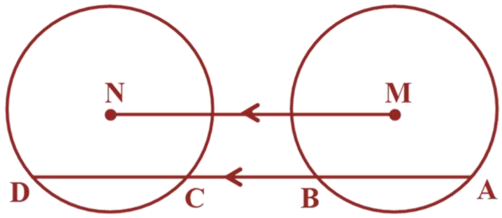
6) MN = zero the two circles are **concentric**



1) In the opposite figure:

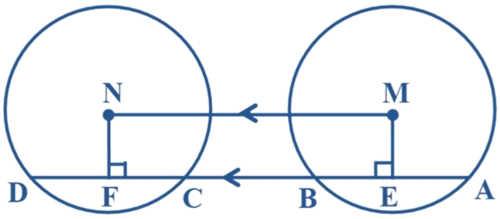
M and N are two congruent circles , AB // MN was drawn and intersects circle M at A and B and intersect circle N at C and D

Prove that : AC = BD



Solution

Construction : Draw $\overline{ME} \perp AD$, $\overline{MF} \perp AD$



$\because$ $\overline{EF} /\!/ \overline{MN}$, m ($\angle$ E) = 90° m ($\angle$ F) = 90°

$\therefore$ $\overline{ME} /\!/ \overline{NF}$

$\therefore$ MENF is a rectangle $\therefore$ $\overline{ME} = \overline{NF}$

$\because$ M and N are two congruent circles $\therefore$ $\overline{AB} = \overline{CD}$

By adding BC to $\overline{AB}$ and $\overline{CD}$

$\therefore$ $\overline{AC} = \overline{ED}$

1) Choose the correct answer :

1) If the surface of the circle M ∩ If the surface of the circle N = ∅ , then the two circles are ……………………

(Distant - touching externally - intersecting)

2) If M and N are two centers of two circles with radii r_1 , r_2 , where MN > r1 + r2 , then the two circles are ……….

(Distant - touching externally - intersecting

3) If the surface of the circle M ∩ If the surface of the circle N = $\{A\}$, then the two circles are ……………………

(touching externally - touching internally - intersecting)

4) If the surface of the circle M ∩ If the surface of the circle N = the surface of the circle N , then the two circles are ……………

(Distant - touching externally - one inside the other)

5) M and N are two circles touching externally , their radii 9cm , 4cm , then MN = …………..cm **(5cm - 7 cm - 13 cm)**

6) M and N are two circles touching internally , their radii 9cm , 4cm , then MN = …………..cm **(5cm - 7 cm - 12 cm)**

7) M and N are two circles, their radii 7cm , 5cm , then MN = 12cm , then the two circles are ……………………..

(Distant - touching externally - touching internally)

8) M and N are two circles, their radii 7cm , 5cm , then MN = 2cm , then the two circles are ……………………..

(Distant - touching externally - touching internally)

9) M and N are two circles, their radii 7cm , 5cm , then MN = 15cm , then the two circles are ……………………

(Distant - touching externally - touching internally)

10) M and N are two circles, their radii 7cm , 2cm , then MN = 3cm , then the two circles are ……………………

(Distant - touching externally - one inside the other)

11) The radius of circle M is 6cm The radius of circle N is 5cm , then MN = 3cm , then the two circles are ……………………

(touching externally - touching internally - intersecting)

12) M and N are two intersecting circles their radii 4cm and 6cm then MN ∈ ………….. **( ]2, 5[,]2, 10[,]4, 9[ )**

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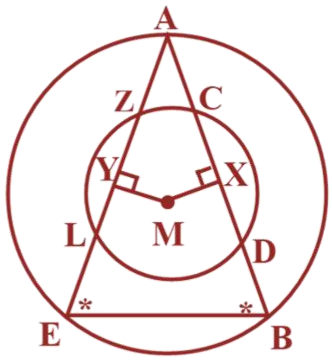
1) In the opposite figure:

Two concentric circles M ,$\overline{AB}$ is a chord in the large circle and intersects the smaller circle at C and D ,$\overline{AE}$ is a chord in the larger circle and intersects the smaller circle at Z and L. if m(∠ABE) = m(∠AEB) then **prove that : CD = ZL**



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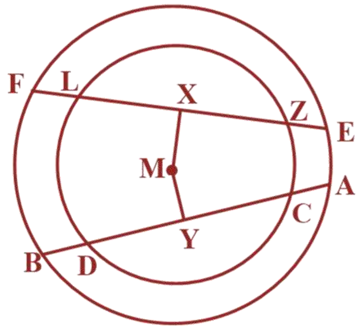
2) In the opposite figure:



Two concentric circles M , $\overline{AB}$ is a chord in the larger circle and intersects smaller circle at C and D. is a chord in the larger circle and intersects the smaller circle at Z and L where AB = EF

Prove that : ***1)*** **CD = ZL** ***2)*** **AD = ZF**

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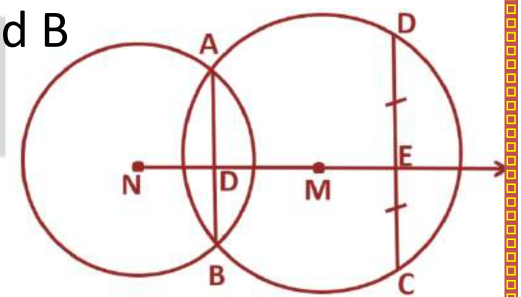
3) In the opposite figure:



The two circles M and N intersects at A and B

CD is a chord in the circle M cuts MN at E

, If E is the midpoint of CD

Prove that $\overline{AB}$ // $\overline{CD}$

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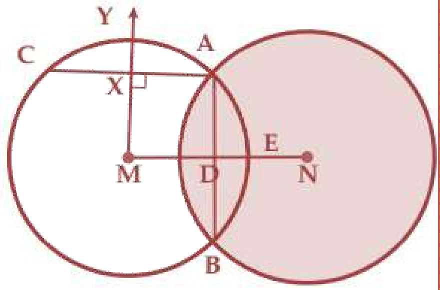
4) In the opposite figure:



The two circles M and N intersect at A and B.

is drawn MX ⊥ AC MN is drawn , AC = AB

***1)* Prove that : MD = MX** ***2)* Prove that : XY = DE**

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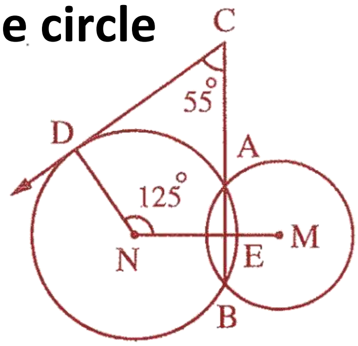
5) In the opposite figure:

M and N are two intersecting circles At A and B , m(∠C)=55° ,

m(∠N)=125° **Prove that : $\overrightarrow{CD}$ is a tangent to the circle**



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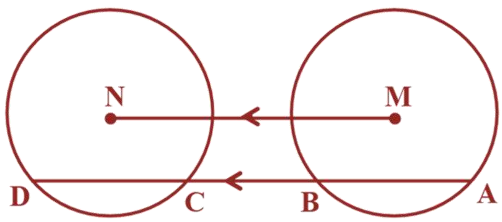
6) In the opposite figure:

M and N are two congruent circles , AB // MN was drawn and intersects circle M at A and B and intersect circle N at C and D

Prove that : A



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REVISION 4

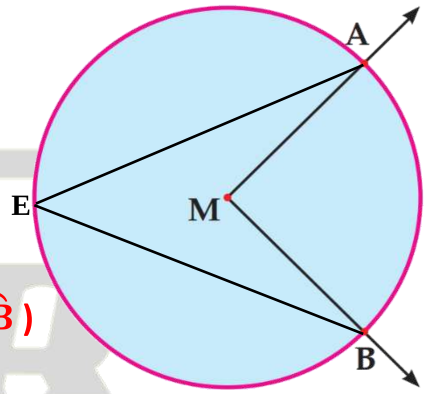
♦ ∠ AMB is called central angle

♦ m($\overset{\frown}{AB}$) = m(∠ AMB)

♦ ∠ AEB is called inscribed angle

♦ m(∠ AEB) = m(∠ AMB) **(subtended by $\overset{\frown}{AB}$)**

♦ m(∠ AEB) = $\frac{1}{2}$ m($\overset{\frown}{AB}$)



♦ **Central Angle** : It is the angle whose vertex is the center of the circle and its sides contain two radii of the circle.

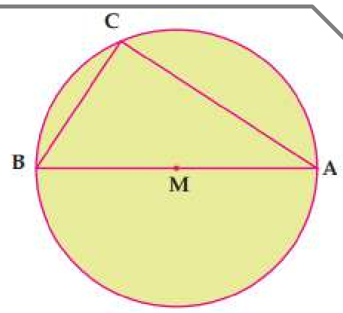
♦ **Measure of the arc** = The measure of the central angle opposite to it.

♦ **Inscribed angle :** An angle the vertex of it lies on the circle and its sides contain two chords of the circle.

♦ **The measure of the inscribed angle** = half the measure of the central angle subtended by the same arc.

♦ **The measure of the inscribed angle** = half the measure of the opposite arc.

♦ The inscribed angle drawn in a semicircle is a right angle.

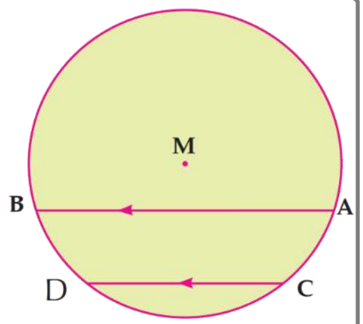


∵ ∠ AEB is drawn in a semicircle

∴ (∠ AEB)=90°

Corollary

♦ If two parallel chords are drawn in a circle, then the measures of the two arcs between them are equal.

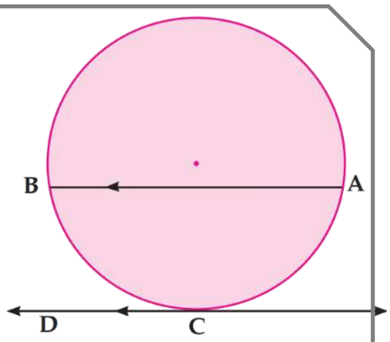


∵ AB // CD

∴ m($\widehat{AC}$) = m($\widehat{BD}$)

Corollary

♦If a chord is parallel to a tangent of a circle, then the measures of the two arcs between them are equal.

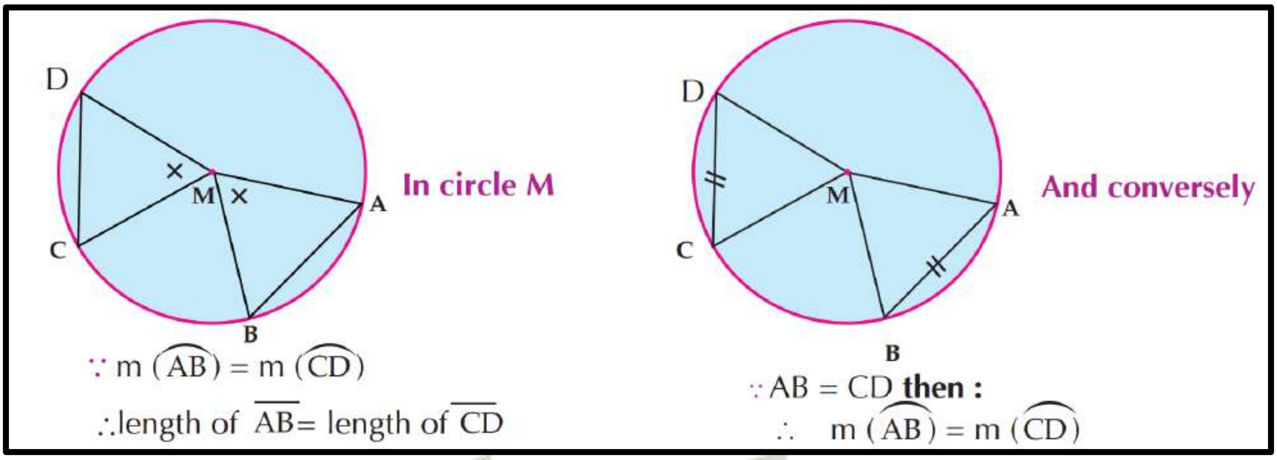


∵ AB // CD

∴ m($\widehat{AC}$) = m($\widehat{BC}$)

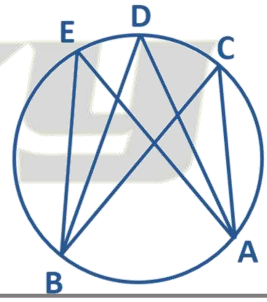
Corollary

♦ If the measures of arcs are equal, then their chords are **equal in length**, and conversely



Theorem

♦ In the same circle, the measures of all inscribed angles subtended by the same arc are equal.

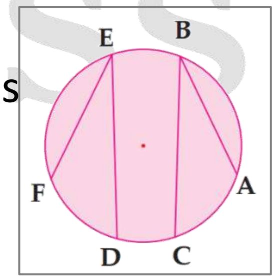


$\because$ ∠ C , ∠ D and ∠ E are inscribed angles subtended by $\overset{\frown}{AB}$

$\therefore\ m(\angle C) = m(\angle D) = m(\angle E)$

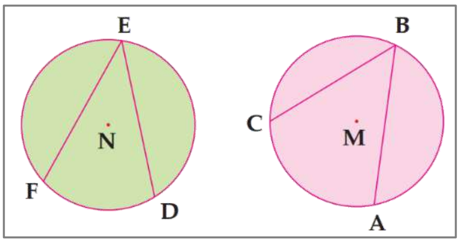
Corollary

♦ In the same circle or in congruent circles, the measures of the inscribed angles subtended by arcs of equal measures are **equal**



$\because$ m($\overset{\frown}{AC}$) = m($\overset{\frown}{FD}$)

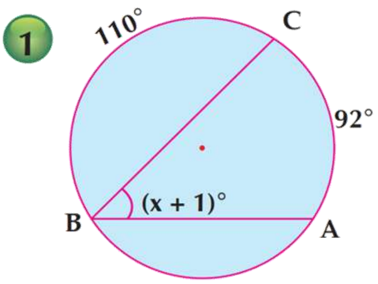
$\therefore\ m(\angle C) = m(\angle D)$



Exercises from school book and governorates' exams

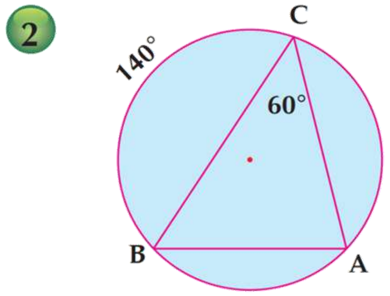
▶ Exercises on the measure of inscribed angle with respect to the measure of arc :

1) Complete the following figures :



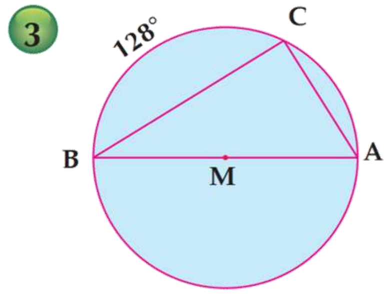
x = ...

m ($\widehat{AB}$) = ...



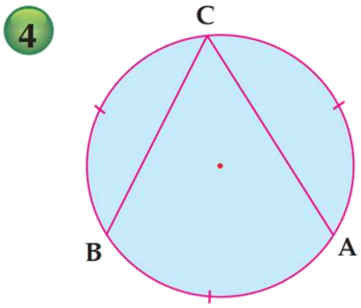
m (∠ A) = ...

m ($\widehat{AC}$) = ...

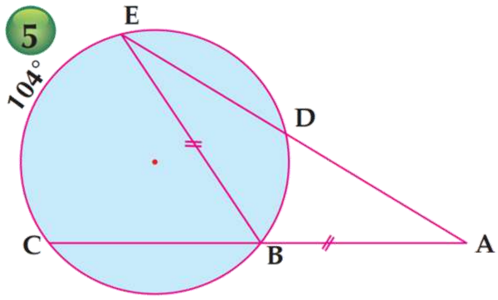


m (∠ C) = ...

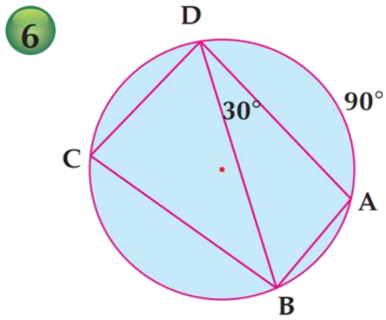
m (∠ B) = ...



m (∠ C) = ...



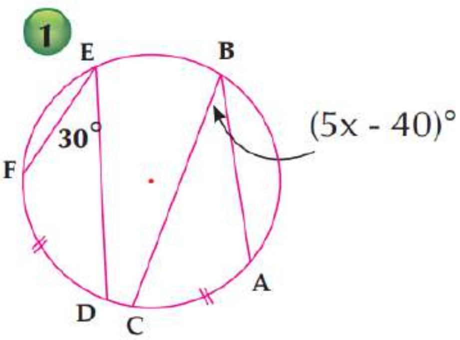
m (∠ EAC) = ...



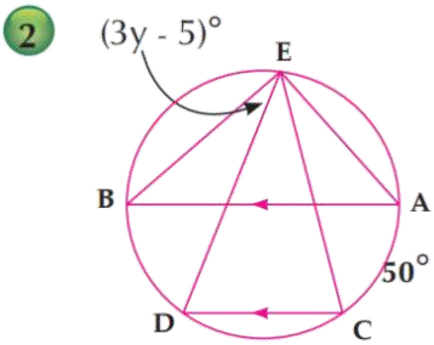
m (∠ DCB) = ...

▶ Exercises on the measure of inscribed angle with respect to the measure of equal arcs :

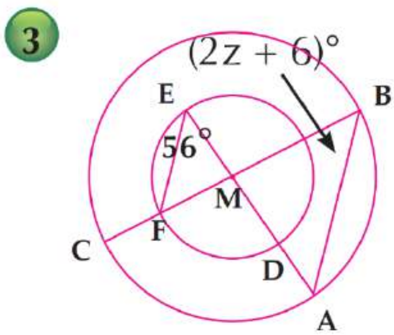
2) find the value of the symbol in the following figures



X = ...



Y = ...

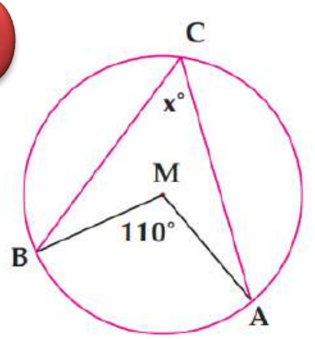


Z =............

▶ Exercises on the measure of inscribed angle with respect to the measure of the central angle :

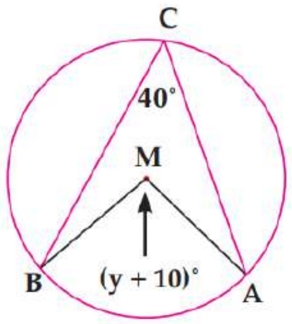
3) find the value of the symbol in the following figures :

1



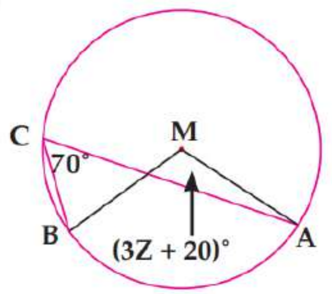
X = ...

2



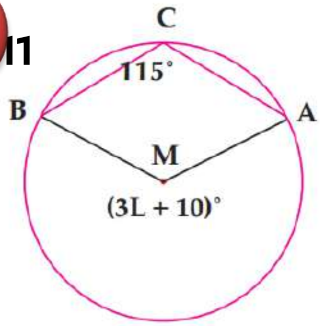
Y = ...

3



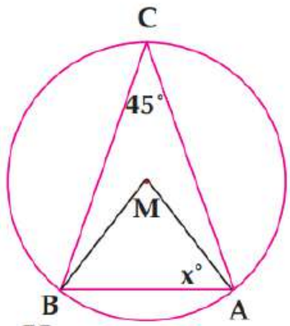
Z = ...

4



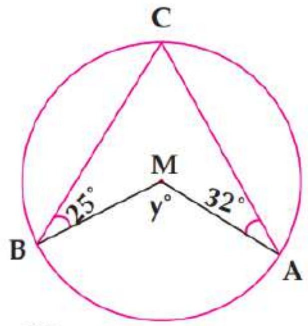
L = ...

5



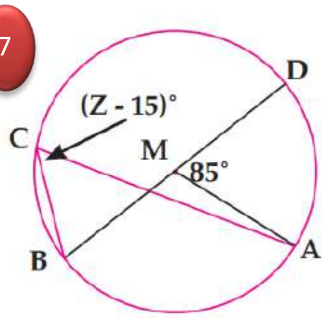
X = ...

6



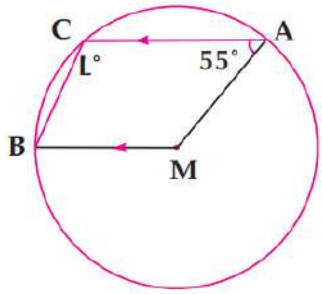
Y = ...

7



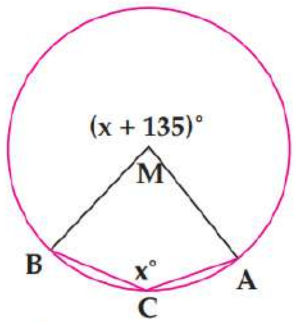
Z = ...

8



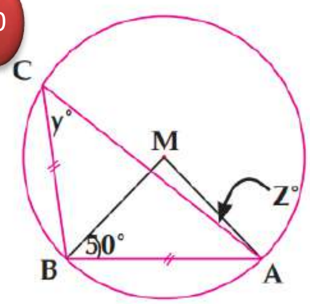
L = ...

9



X = ...

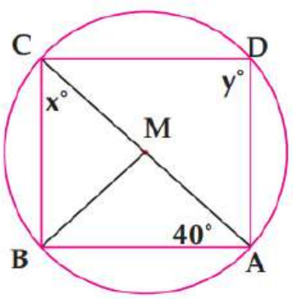
10



Y = ...

Z = ...

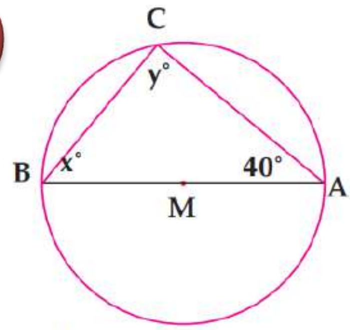
11



X = ...

Y = ...

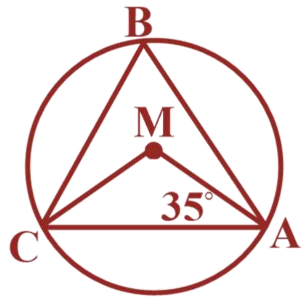
12



X = ...

Y = ...

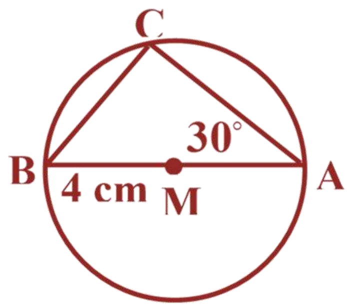
4) In the opposite figure:



M is a circle , m (∠ MAC) = 35°

Find m (∠ ABC)

5) In the opposite figure:



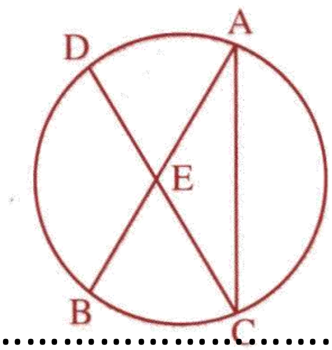
$\overline{AB}$ is a diameter in the circle M

with radius length 4 cm , m (∠ A) = 30°

1) Find m (∠ ABC)

2) Find the length of BC

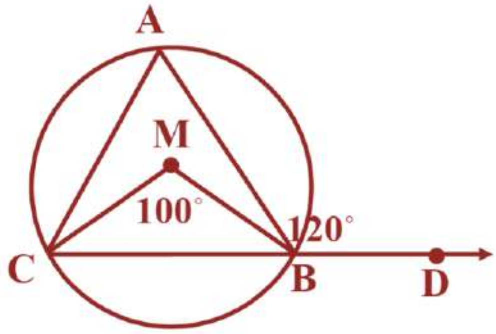
6) In the opposite figure:



AB and CD are two equal chords

Prove that Δ AEC is isosceles

7) In the opposite figure:



Δ ABC drawn in the circle M

D $\in \overrightarrow{CB}$ such that m ($\angle$ ABD)=120°

if m ($\angle$ BMC)=100°

Find with proof m ($\angle$ ACB)

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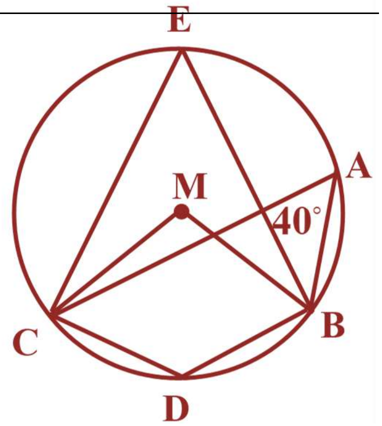
8) In the opposite figure:



The chords $\overline{AC}$ and $\overline{BE}$ intersects

At X , M is the centre of the circle,

if m($\angle$BAC)=40°

Find:

1) m($\angle$BEC)   2) m($\angle$BMC)   3) m(BDC)

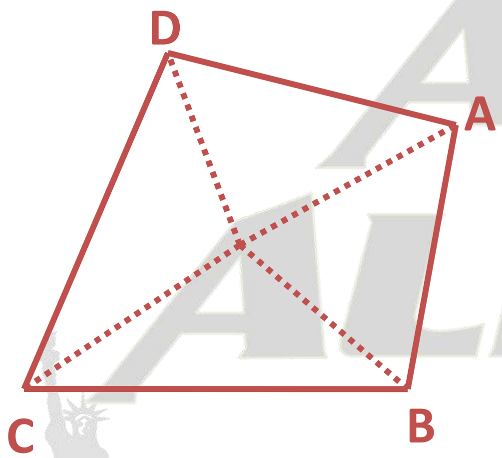
The quadrilateral is a cyclic if one of the following conditions is verified:

1) If there is a point in the plane of the figure such that it is equidistant from its vertices.

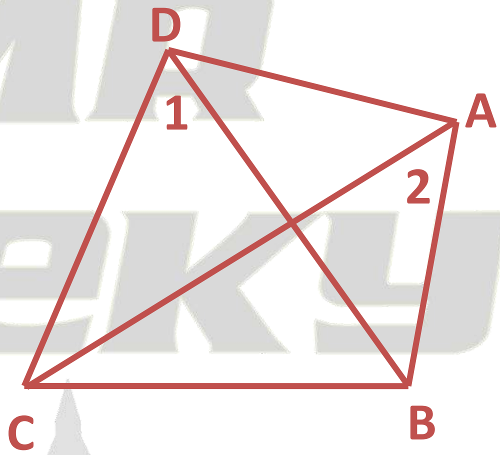
2) If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.

3) If there are two opposite supplementary angles " their sum =180"

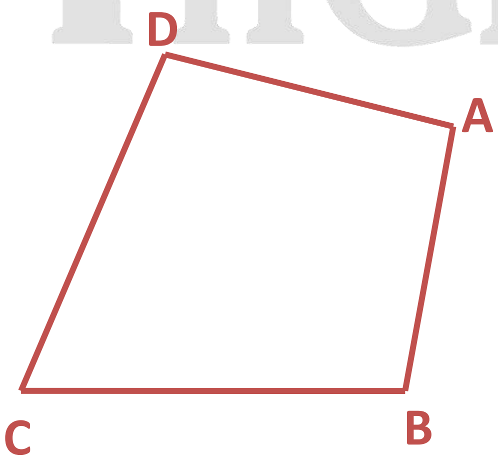
4) If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.



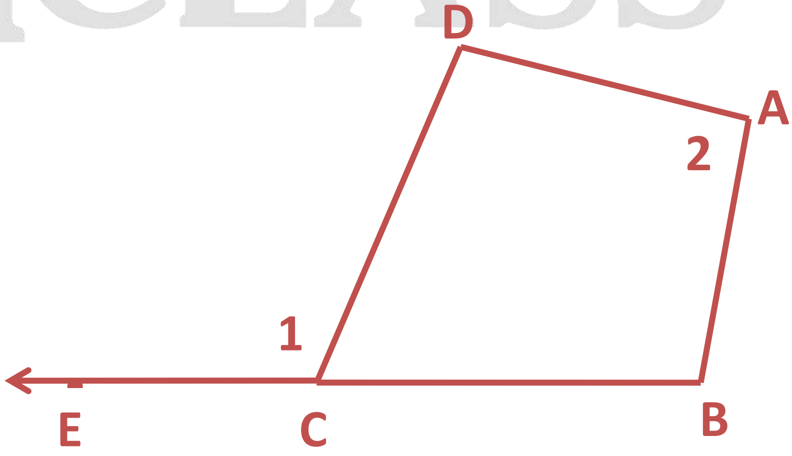
$$MA = MB = MC = MD$$



$$m(\angle 1) = m(\angle 2)$$

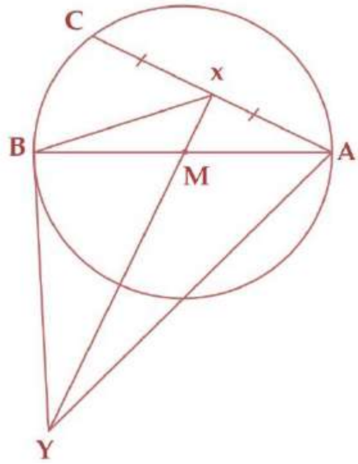


$$m(\angle A) + m(\angle C) = 180^\circ$$



$$m(\angle 1) = m(\angle 2)$$

In the opposite figure:



AB is a diameter in circle M, X is the midpoint of AC and XM intersecting the tangent of the circle at B in Y .

Prove that :

The figure AXBY is a cyclic quadrilateral.

solution

∵ X is the midpoint of AC ∴ MX ⊥ AC, m (∠ AXY) = 90°

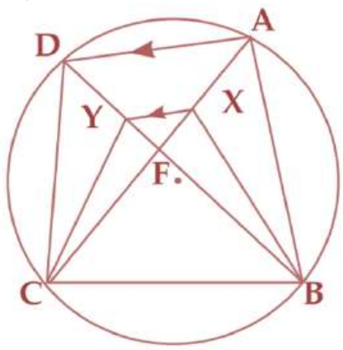
∵ AB is a diameter and , BY is a tangent at B

∴ BY ⊥ AB , m (∠ ABY) = 90°

∵ m (∠ AXY) = m (∠ ABY) = 90°

∴ Figure AXBY is a cyclic quadrilateral.

In the opposite figure:



ABCD is a cyclic quadrilateral with diagonals intersecting at F, X ∈ AF and Y ∈ DF where XY // AD .

Prove that :

First : BXYC is cyclic quadrilateral.

Second : m(∠XBY) = m(∠XCY)

solution

∵ XY//AD ∴ m (∠CAD) = m (∠CXY) Corresponding

∵ m (∠CAD) = m (∠CBD) both are inscribed and common in $\overline{CD}$

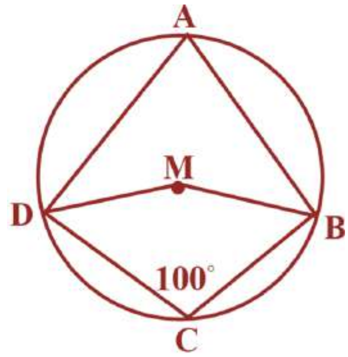
∴ m (∠CXY) = m (∠CBY) (two inscribed angles on the base $\overline{CY}$)

∴ BXYC is a cyclic quadrilateral

∵ BXYC is a cyclic quadrilateral ∴ m (∠XBY) = m (∠XCY)

because they are both inscribed angles common at $\widehat{CD}$

1) In the opposite figure:

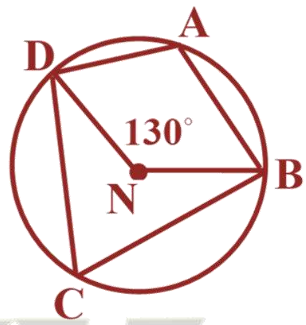


M is a circle ABCD is a cyclic quadrilateral ,

m (∠ C) = 100°

Find : *1)* $m(\angle A)$ *2)* $m(\overset{\frown}{BCD})$

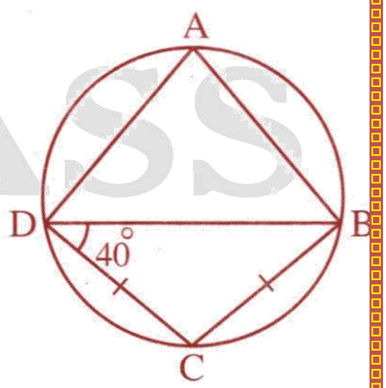
2) In the opposite figure:



ABCD is a quadrilateral drawn
in the circle N , if m (∠BND) = 130°

Find : $m(\angle BAD)$

3) In the opposite figure:

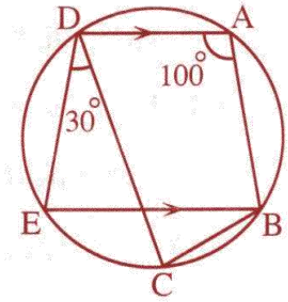


ABCD is a cyclic quadrilateral ,

m (∠ CDB) = 40° , BC = DC

Find : $m(\angle A)$

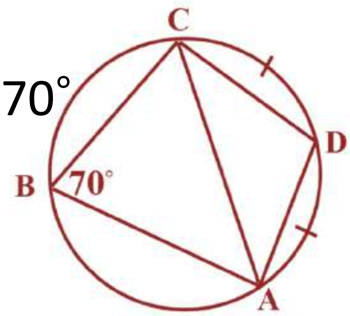
4) In the opposite figure:



$\overline{AD} \parallel \overline{BE}$ **,** m (∠BAD)=100°

m (∠EDC)=30° **,** **Find :** m (∠CDA)

5) In the opposite figure:

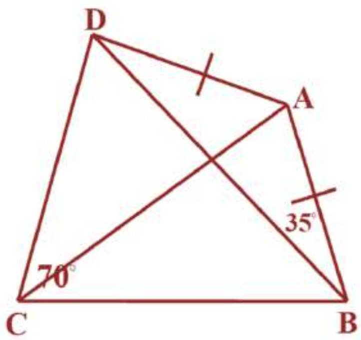


ABCD is a cyclic quadrilateral in which m(∠ABC)=70°

The length of $\overset{\frown}{AD}$ = The length of $\overset{\frown}{DC}$

Find : m(∠ACD)

6) In the opposite figure:



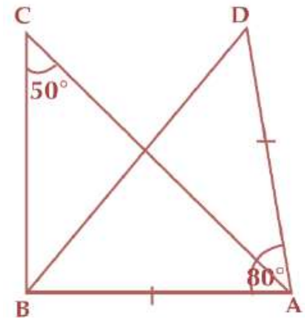
ABCD is a quadrilateral , where AB = AD ,

m(∠ABD) = 35°, $m(\angle BCD) = 70°$

Prove that:

1) ABCD is a cyclic quadrilateral
2) 2) $\overrightarrow{CA}$ bisects ∠BCD

7) In the opposite figure:



AB = AD , m (∠ A) = 80° , m (∠ C) = 50°

Prove that : The points A, B, C and D have one circle passing through them.

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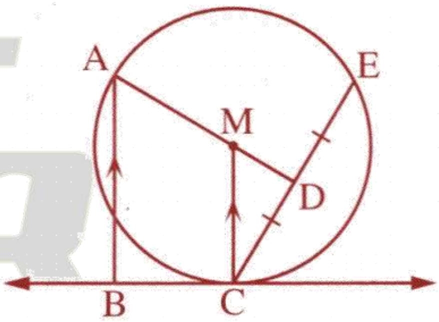
8) In the opposite figure:



$\overleftrightarrow{BC}$ is a tangent to the circle M at C ,

D is the midpoint of EC , $\overline{MC} \parallel \overline{AB}$

Prove that :

ABCD is cyclic quadrilateral .

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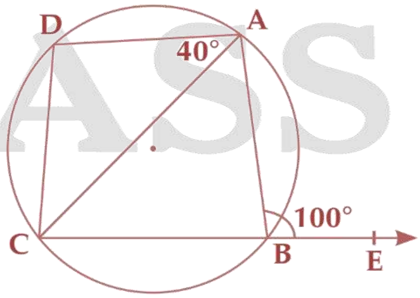
9) In the opposite figure:



m (∠ ABE) = 100° , m (∠CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) .

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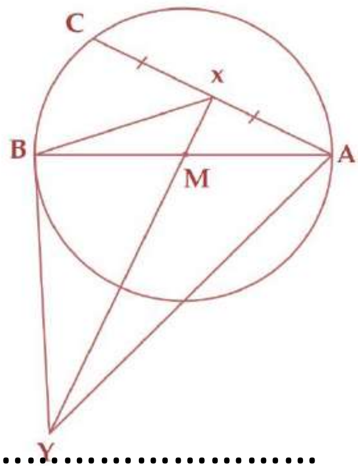
10) In the opposite figure:

AB is a diameter in circle M, X is the midpoint of AC and XM intersecting the tangent of the circle at B in Y .



Prove that :

the figure AXBY is a cyclic quadrilateral.

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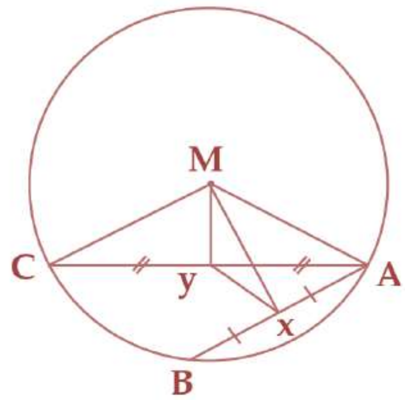
11) In the opposite figure:

A circle with center M. X and Y are the two midpoints of AB and AC respectively.



Prove that :

First : AXYM is a cyclic quadrilateral.

Second : m (∠MXY) = m (∠MCY)

Third : AM is a diameter in the circle passing through the points A, X, Y and M

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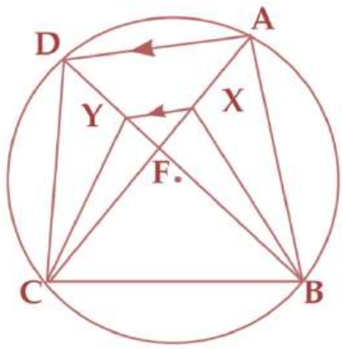
12) In the opposite figure:



ABCD is a cyclic quadrilateral with diagonals intersecting at F, X ∈ AF and Y ∈ DF where XY // AD .

Prove that :

First : BXYC is cyclic quadrilateral.

Second : m(∠XBY) = m(∠XCY)

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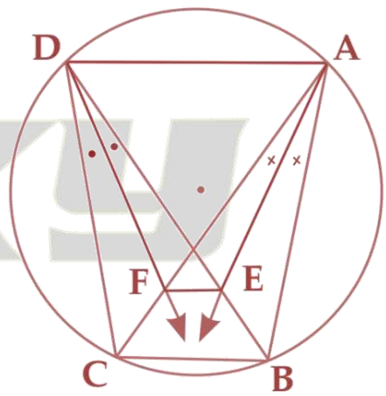
13) In the opposite figure:



In the opposite figure : ABCD is a cyclic quadrilateral which has $\overrightarrow{AE}$ bisects ∠BAC and $\overrightarrow{DF}$ bisects ∠BDC , **Prove that :**

First: AEFD is a cyclic quadrilateral

Second: $\overline{EF}$ // $\overline{BC}$.

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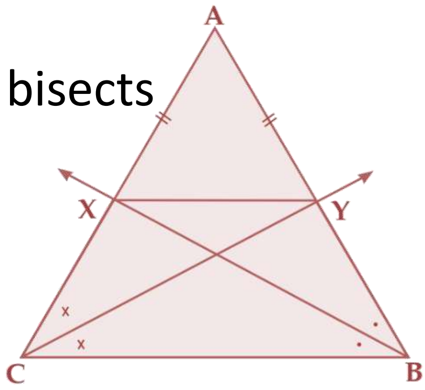
14) In the opposite figure:



ABC is a triangle in which has AB = AC and $\overrightarrow{BX}$ bisects ∠Band intersect AC at X , $\overrightarrow{CY}$ bisects ∠C and intersect AB at Y , Prove that :

First: BCXY is a cyclic quadrilateral.

Second: $\overline{XY}$ // $\overline{BC}$

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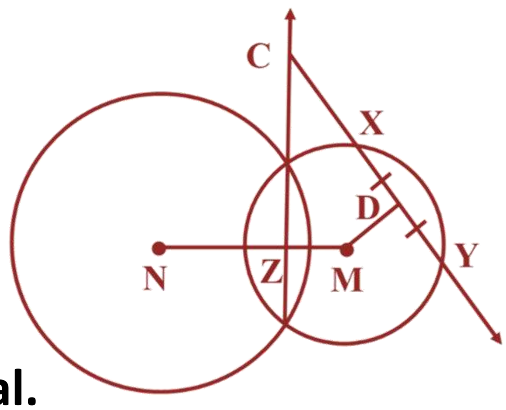
15) In the opposite figure:



Circle M ∩ Circle N = { A , B }

C ∈ BA and C ∉ $\overline{BA}$ draw CX

to cut circle M at X And Y if D is

the midpoint of $\overline{XY}$ and $\overline{AB}$∩$\overline{MN}$={Z}

Prove that : CDMZ is a cyclic quadrilateral.

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